Definite Descriptions and Choice Functions


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1. Introduction

The analysis of definite descriptions is not only an important issue in philosophy, philosophy of language, logic and foundations of mathematics but it is also of central interest to linguistic theory. Definite descriptions are a major issue for semantic research because they play a prominent role among the referring terms in a language. Referring terms, like proper names, definite noun phrases and indexical or demonstrative expressions, are the essential tools of a language to connect the meaning of linguistic expressions with objects in the real world, or a discourse world. Proper names refer to their referents by conventional rules, demonstratives fix their referents due to an ostension and indexical expressions determine their referents according to situational or pragmatic information. Definite descriptions show a more complex behavior, since they refer to their objects through descriptive or lexical information, on one hand, and contextual or pragmatic information, on the other hand. They form a continuum from exclusively lexically determined expressions to situationally dependent expressions. Therefore, an analysis of definite descriptions has to cope with this two-sided character, which can be found in other linguistic expressions, as well. In connection with this problem, it is very controversial whether definiteness is a semantic principle or a pragmatic strategy. But even if it is a pragmatic principle definiteness is essential for the interpretation of linguistic expressions. For the representation of definite descriptions, one has to decide to encode definiteness in the semantic structure of the expression or leave it outside. In this case, definiteness has to be included in the pragmatic rules of application. And finally, the analysis of anaphora is intertwined with the representation of definiteness and definite descriptions. These are only few aspects of definite descriptions that have caused the special interest in linguistics investigating their nature.

The concept of definite descriptions was created and discussed in philosophy and epistemology. There are at least two types of linguistic expressions that correspond to the philosophical concept of definite descriptions: (i) definite noun phrases or noun phrases with the definite article like the first man on the moon, the pope, the man at the corner, the table etc. and (ii) anaphorical pronouns which do not stand in the scope of their antecedents: A man comes. He whistles. These two types of expressions are not always investigated together. Definite noun phrases are regarded as the typical instance of definite descriptions, whereas the status of anaphorical pronouns or discourse anaphora is controversial. But even approaches that analyse such pronouns as definite descriptions make important differences between the analysis of
definite noun phrases as definite descriptions on the one hand, and discourse
pronouns as definite descriptions on the other hand. I will try to follow each separate
line of argumentation, and finally merge them together to a uniform analysis of
definite descriptions in linguistic theory. The example in (1) should illustrate the
narrow connection between the analyses of both types of expression:

(1a) The man whistles.
(1b) A man comes. The man whistles.
(1c) A man comes. He whistles.

In (1a) the definite noun phrase the man must be interpreted according to the non-
linguistic context, whereas the same definite noun phrase in (1b) is anaphorically
linked to the indefinite noun phrase a man in the antecedent clause. In (1c) the
anaphorical pronoun has the same anaphorical function as the definite noun phrase
in (1b) and it refers to the same object. A linguistic theory should be able to describe
these phenomena uniformly.

The organization of the paper is as follows: In the second section, I introduce
Russell’s classical theory of descriptions and two of its applications to linguistic
theory. First, ambiguities of definite descriptions can be represented as scope
interactions with other operators. And second, the representation of discourse
pronouns as definite descriptions and not as bound variables allows a new
perspective on the nature of anaphora. Successful as it might be, the Russellian
analysis has serious shortcomings in both areas of application that calls for a new
analysis of definite descriptions. In section 3, I discuss the alternative approach in
which definite descriptions are interpreted with choice functions. This analysis goes
back the metamathematical investigations of Hilbert and Bernays, but it was rarely
used in linguistic theory, which shows a conceptual insecurity in the nature of choice
functions. There are quite different uses for choice functions in semantics that cover
the representation of indefinite noun phrases, definite noun phrases and anaphorical
pronouns. Most of these applications were not generally accepted because of the
indeterminacy of the concept of choice functions. In section 4, I propose a elaboration
of this concept: context dependent choice functions. This modification reconstructs
the idea of a context dependent salience hierarchy according to which definite
descriptions get their referents. In this modified representation, definite noun
phrases and discourse anaphora can be uniformly represented. The move towards
using context dependent choice functions raises a new important issue: How do
linguistic expressions change a given salience hierarchy, and how can this salience
change potential be built into the semantics? Section 5 proposes an additional level of
meaning that represents this salience change potential, and its interaction with the
interpretation of linguistic expression.

2. Russell’s classical theory of descriptions
Russell developed a new representation for definite descriptions for a metamathematical
analysis in his Principia Mathematica. He also provided philosophical and
epistemological motivations to use his approach in natural language analysis, and it
became the common semantic theory of definite descriptions in natural language.
The most important feature of Russell’s theory is that he does not regard definite
descriptions to be referring terms like proper names or indexical expression, but to be

definite noun phrases as definite descriptions on the one hand, and discourse
pronouns as definite descriptions on the other hand. I will try to follow each separate
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The most important feature of Russell’s theory is that he does not regard definite
descriptions to be referring terms like proper names or indexical expression, but to be
denoting phrases like quantifier phrases. Definite descriptions do not refer to their referents directly like referring expressions, but they describe their referents in a unique way in the context of a sentence. Therefore, Russell gives no independent interpretation of definite descriptions but he only accounts for an interpretation in an atomic sentence:

(2) The first man on the moon was an American.

(2a) \( \exists x \ [\text{First\_man\_on\_the\_moon}(x) \& \forall y \ [\text{First\_man\_on\_the\_moon}(y) \leftrightarrow x = y] \& \text{American}(x)] \) (predication)

Sentence (2) with the definite description \textit{the first man on the moon} gets the representation (2a) that consists of the conjunction of three clauses which express existence, uniqueness and matrix predication. This can be abbreviated to the more familiar form in (2b), with the iota operator as a representation for the definite article pretending to reproduce the functor argument structure of the natural sentence. Russell (1905) states the equivalence of (2a) and (2b), which is a context definition since it defines the iota term only in the context of an atomic sentence. The existence clause guarantees that the set which is denoted by the descriptive content of the definite descriptions (here: \textit{first man on the moon}) is not empty. Sentences with empty descriptions like \textit{the flying horse} cannot be true because they do not fulfill this existence condition. The uniqueness clause determines that there is only one object with the expressed property. It distinguishes indefinite descriptions from definite ones. However, this is the most problematic part of Russell’s theory from a linguistic point of view, as we see below. Russell’s theory expresses the existence and the uniqueness of an object that falls under the descriptive content in the semantic representation. Thus, definiteness is included in the semantic structure of linguistic expressions. Russell’s classical theory is the standard view among linguists even today (cf. Heim 1991). The two main applications of this theory are with regard to definite noun phrases and E-type pronouns.

2.1 Definite noun phrases as iota terms

The Russellian analysis for definite descriptions serves as the standard interpretation of definite noun phrases. The alternative position consists of the assumption that definite noun phrases are referring expressions like proper names, or indexical expressions. The two most important arguments in favour of Russell’s theory concern scope and the complex structure of definite noun phrases.\(^1\) The context definition of definite descriptions creates different scope readings of the definite expression regarding other quantifiers. For Russell, this was an unwelcome result of his theory, which can be demonstrated on the behavior of negation.

(3) The first man in space was not an American.

(3a) \( \neg \text{American}(\iota x \ \text{First\_man\_in\_space}(x)) \)

(3b) \( \neg \exists x \ [\text{First\_man\_in\_space}(x) \& \forall y \ [\text{First\_man\_in\_space}(y) \leftrightarrow x = y] \& \text{American}(x)] \)

(3c) \( \exists x \ [\text{First\_man\_in\_space}(x) \& \forall y \ [\text{First\_man\_in\_space}(y) \leftrightarrow x = y] \& \neg \text{American}(x)] \)

The logical form (3a) of the sentence in (3) is not a proper form since the iota term is
an abbreviation for a conjunction of three clauses. According to Russell, through the contextual definition of the iota operator we can derive the two (proper) forms (3b) and (3c). The two forms have the same truth conditions as long as the existence and uniqueness conditions are fulfilled. However, for empty descriptions the two forms amount to different truth conditions. Given the situation in which no man has been in space, form (3b) would become true because the existence of such an individual is negated. But (3c) would falsely assert that there is such a man, who is not an American. Therefore, Russell stated that definite descriptions always have narrow scope regarding other quantifiers.

In contrast to Russell’s narrow scope analysis, there are several linguistic contexts in which an analysis with the original scope ambiguity is preferred. The following example illustrates this fact:

\[
\text{(4a) } \forall x \left[ \text{President}_\text{of}_\text{the}_\text{USA}(x, t) \rightarrow \text{White}(x) \right]
\]

The two formulas (4b) and (4c) can be derived from the quasi representation (4a) of (4) with the operator \(F \text{ Always}(t)\) for \text{will always}. The operator binds a time parameter \(t\) on which the definite noun phrase depends. They represent two different readings of the sentence (4): In one reading, the sentence means that for all times the president (at that time) will be white. This may be falsified by at least one black person who would become president. This reading is represented by the form (4b). The other reading means that the present president will be always white. This could only be falsified if the person changed his or her color. As in the form (4c), the definite noun phrase has wide scope over the time operator. Interactions with other quantifiers are possible, as well (cf. Neale 1990).

Another advantage of the Russellian representation of definite noun phrases is the possibility to make definite noun phrases dependent on other parameters, e.g. on the time parameter in (4) or on the universal quantifier as in (5):

\[
\text{(5a) } \forall x \left[ \text{Man}(x) \rightarrow \text{Love}(x, \text{ι} y) \left[ \text{Woman}(y) \& \text{Raised}(y, x) \right] \right]
\]

The quasi form (5a) of sentence (5) can be decomposed into (5b) or (5c). But only (5b) represents the meaning of (5), since in (5c) the variable \(x\) in the existential clause is not bound. Thus the dependency on the universal quantifier forces the narrow scope reading (5b) of the iota term regarding the universal quantifier. This corresponds to the linguistic intuition that (5) possesses only one reading (in which the pronoun \textit{him} is linked to the quantifier phrase). Because of these and other features, the Russellian theory of descriptions serves as the most prominent analysis for definite noun phrases (cf. Heim 1991).

However, the Russellian uniqueness claim is a very strong assumption that seems not to be warranted by linguistic data. The analysis works only if the descriptive
content of the noun phrases is a unique like *first man in space* or a functional concept like *the father of.* Definite expressions with sortal concepts like in (6) or (7) are problematic for the analysis.

(6) The table is covered with books.
(7) A man walks. The man whistles.

Expressions like *the table* in (6) are also called indefinite or incomplete definite noun phrases. The definite noun phrase *the man* in (7) is anaphorical linked to indefinite noun phrase *a man* in the antecedent sentence. It refers to one particular man, even if there are more than one man in the discourse. Common occurrences of definite noun phrases seem to be of these types. Russell’s theory can cope with them only by introducing additional modifications and restrictions on the domain of interpretation. However, we will see in section 2.2 that even such restriction do not work in all cases.

### 2.2 E-type pronouns as iota terms

The second important application of Russell’s theory consists of the representation of E-type pronouns. The nature of anaphorical relations and their representation is controversial even today. Sentence internal anaphorical pronouns are generally represented as bound variables like in predicate logic. The pronoun *him* in (5) is represented as the variable $x$ that is bound by the quantifier phrase. Intersentential anaphorical pronouns, or E-type pronouns, cause additional problems because they cannot simply be represented as bound variables: they stand outside of the syntactic scope of the relevant operator. The pronoun *he* in (8) is represented as the variable $x$ in (8a). This variable cannot be bound by the existential quantifier, instead, it remains free.

(8) A man comes. He whistles.
(8a) $\exists x [\text{Man}(x) \& \text{Come}(x) \& \text{Whistle}(x)]$

There are essentially three approaches to solve this problem: First, the concept of binding is extended so that the existential quantifier can bind the variable. This view goes back to Geach (1962) and was developed first by Heim (1982) and Kamp (1981) in different types of a discourse representation theory, where the representation of (8) looks like (8b). This approach, finally, lead to so-called dynamic semantics (e.g. Groenendijk & Stokhof 1991), which give a dynamic interpretation to the existential quantifier and the conjunction such that the equivalence between (8c) and (8d) holds.

I cannot give a satisfying discussion of the binding approach here.

(8b) $\{d_1 \mid \text{Man}(d_1) \& \text{Come}(d_1) \& \text{Whistle}(d_1)\}$
(8c) $\exists_{\text{dy}n} x [\text{Man}(x) \&_{\text{dy}n} \text{Come}(x) \&_{\text{dy}n} \text{Whistle}(x)]$
(8d) $\exists_{\text{dy}n} x [\text{Man}(x) \&_{\text{dy}n} \text{Come}(x) \&_{\text{dy}n} \text{Whistle}(x)]$

An alternative approach consists of representing the anaphorical pronoun as a definite description. This idea is based on the work of Evans (1977) and Cooper (1979), and was newly applied in Heim (1990), Neale (1990), Chierchia (1992) and von der Does (1993). The theory is also known as E-type theory, a name that was created by Evans (1977). This approach can keep to classical static predicate logic and represent sentence (8a) by the logical form (8e) in which the iota term stands for the E-type pronoun.

(8e) $\exists x [\text{Man}(x) \& \text{Come}(x)] \& \text{Whistle} (\iota x [\text{Man}(x) \& \text{Come}(x)])$
A third alternative was noted by Kripke (1977) and Lewis (1979), who regard the
anaphorical pronoun as an expression that refers to the most salient object in
discourse. This view seems to fit the intuitions of the traditional grammarians, as
well (e.g., Jespersen 1925). Most attempts to formalize this idea use epsilon terms
representing the pronoun. The epsilon term (8f) refers to the most salient object
which fits the descriptive content. We will discuss this approach in some detail in
section 3.4.

\(8f\) \(\exists x [\text{Man}(x) & \text{Come}(x)] \& \text{Whistle}(\varepsilon x [\text{Man}(x) & \text{Come}(x)])\)

In the reminder of this section, I must confine the discussion to the E-type
approach. On the one hand, the E-type approach seems to be quite adequate for
representing discourse anaphora, on the other hand, it has some problems, of which
the two most serious are the following: the rule that recovers the descriptive material
of the definite descriptions standing for the pronouns and the uniqueness condition
of the Russellian interpretation of the definite description. The first problem concerns
the rule according to which the descriptive content of the definite noun phrase is
reconstructed. Neale (1990, 182) states two rules differentiating between whether the
antecedent expression is a definite noun phrase or an indefinite noun phrase.

(9) A man comes. He whistles.
(9a) The man who comes whistles.
(9b) \(\exists x [\text{Man}(x) & \text{Come}(x)] \& \text{Whistle}(\iota x [\text{Man}(x) & \text{Come}(x)])\)

(10) The man comes. He whistles.
(10a) The man whistles.
(10b) \(\text{Come}(\iota x \text{Man}(x)) \& \text{Whistle}(\iota x \text{Man}(x))\)

The anaphorical pronoun he in (9) can be rephrased as the definite description the
man who comes in (9a). Its descriptive material is recovered from the antecedent
expression and the matrix sentence in which the antecedent stands. If, however, the
antecedent is itself a definite noun phrase, the definite description representing the
anaphorical pronoun is reconstructed only from the antecedent like in (10). It is
questionable what kind of rule this represents: a heuristic, a pragmatic strategy or a
linguistic rule.2

The second and more serious problem of this approach lies in the uniqueness
condition of the Russellian analysis. This point can be illustrated by the following
eexample (Heim 1982, 28):

(11a) A wine glass broke last night. It had been very expensive.
(11b) The wine glass that broke last night had been very expensive
(11c) Just one wine glass broke last night.

The pronoun it in (11a) is represented by the definite description the wine glass that
broke last night as in (11b). However, the Russellian analysis of definite descriptions
claims (11c), which is a too strong condition. For sentence (11a) could also be uttered
in a situation where more than one wine glass might have been broken only some of
which were expensive. The same line of argument is also taken in the rejection of the
E-type analysis of donkey sentences as in (12):

(12) Every farmer who owns a donkey beats it.
(12a) For every farmer x and for every donkey y, if x owns y then x beats y.
(12b) \(\forall x \forall y [(\text{Farmer}(x) & \text{Donkey}(y) & \text{Owns}(x, y)) \rightarrow \text{Beats}(x, y)]\)
(12c) Every farmer who owns a donkey beats the (unique) donkey he owns.
(12d) \(\forall x [(\text{Farmer}(x) & \exists y \text{Donkey}(y) & \text{Owns}(x, y))\)
The intuitive standard (universal) reading of the donkey sentence in (12) can be captured by the paraphrase in (12a). The standard notation is (12b), where the indefinite noun phrase a donkey gets universal force. The E-type analysis (12c) and (12d) can only assign one donkey for each farmer because of the Russellian semantics. This, however, does not meet the intuitive meaning of the sentence (12), since the sentence would already be false if a farmer owns more than one donkey (regardless whether he beats it or not). The problem of the uniqueness condition is the biggest obstacle for using the E-type analysis (cf. Heim 1982, Heim 1990). It is interesting to note, that the problems with uniqueness serve as a strong argument against the analysis of E-type pronouns as definite descriptions, but the same argument is rejected if it is directed against the analysis of definite noun phrases as Russellian descriptions. Thus we can state the following: the analysis of definite noun phrases and discourse anaphora as definite descriptions is quite successful. Still, the Russellian semantics of definite descriptions contains conditions which are too strong for natural language description.

3. Choice functions and linguistic theory
The idea to replace the Russellian uniqueness condition for definite descriptions with choice functions goes back to Hilbert and Bernays (1939). Choice functions assign one of its elements to each non-empty set, and an arbitrary element to the empty set. Still, choice functions did not come to any prominence in linguistic theory for various reasons. Firstly, Hilbert and Bernays developed the epsilon operator - the syntactic counterpart of choice functions - for definite expression in metamathematical reasoning. Therefore, the epsilon is not primarily designed for natural language descriptions. Secondly, other than Russell, they did not give any motivation for using the epsilon in epistemology or linguistics. Thirdly, the indeterminacy of the epsilon makes it difficult to determine a clear field of application besides the domain of natural numbers. The uncertainty of the character of the epsilon becomes obvious in the multiple applications of the epsilon that are discussed in section 3.2 -3.4. And finally, a static view on the meaning of linguistic expression prevents the use of the epsilon in linguistic theory. We will see in section 4 that only the dynamicization of meaning allows it to employ the epsilon operator in a sensible way.

3.1 Hilbert's epsilon
Hilbert and Bernays introduced the epsilon operator to replace their version of the Russellian iota operator in order to represent definite expressions and to eliminate quantifiers in metamathematical proofs. The epsilon operator can be seen as a generalized iota operator without existence and uniqueness conditions. Hilbert introduced the epsilon operator as an indeterminate symbol that assigns an element to a given set, but does not specify which element. Thus an epsilon term is a constant that stands for a set by referring to an arbitrarily chosen element of this set. The advantage of assuming an indeterminate symbol is that the universal and the existential quantifier can be defined by the epsilon operator, and we can therefore derive a quantifier-free equivalent for each formula.

Hilbert and Bernays did not give an explicit semantics for the epsilon because they
used the epsilon only as an auxiliary symbol for proof-theoretical purposes. At the end of a proof the epsilon is eliminated. Other systems use the epsilon as a basic symbol that has a formation rule and an interpretation. The syntactic characterization consists of the following three conditions: (13) expresses the introduction rule for epsilon terms, (14) formulates the principle of substitutivity and (15) describes the extensionality. The epsilon formula in (13) entails the two Hilbert rules (16) and (17). The second Hilbert rule can also derived from the first one by substituting $\neg F$ for $F$ and by contraposition.

$\exists x Fx \rightarrow F \varepsilon x Fx$ must be valid.

Every expression of the form $\varepsilon x Fx$ must get a value such that all variables can be substituted by epsilon terms.

$\forall x [Fx \leftrightarrow Gx] \rightarrow \varepsilon x Fx = \varepsilon x Gx$ must be valid.

$\exists x Fx \equiv F \varepsilon x Fx$ first Hilbert rule

$\forall x Fx \equiv F \varepsilon x \neg Fx$ second Hilbert rule

Hilbert and Bernays did not account for the semantics of their epsilon symbol, leaving this task for others. Schröter (1956) proposed interpreting the epsilon operator through a choice function. Asser (1957) then formulated this idea in the necessary detail. Following Asser, we will interpret the epsilon operator with a choice function $\Phi$, which assigns one of its elements to each non-empty set, and an arbitrary element to the empty set. For the model-theoretic interpretation we have to extend a model $M = <D, I>$ with the choice function $\Phi$ to $M = <D, I, \Phi>$. The epsilon term $\varepsilon x Fx$ is interpreted as that object which the choice function $\Phi$ assigns to the extension of $F$.

$[[\varepsilon x Fx]]^M_{\Phi} = \Phi([[F]]^M_{\Phi})$ where $\Phi$ is a choice function that is determined by the model $M$.

$\Phi([[F]]^M_{\Phi}) \in [[F]]^M_{\Phi}$ if $[[F]]^M_{\Phi} \neq \emptyset$

$\Phi([[F]]^M_{\Phi}) \in D$ if $[[F]]^M_{\Phi} = \emptyset$

Before I discuss three different applications of Hilbert’s epsilon in linguistic theory I emphasize its two-sided character. On the one hand, the epsilon refers arbitrarily to an object of the given set. This indeterminate choice makes an epsilon term in a certain way indefinite. On the other hand, the epsilon assigns to a given set always the same object. Thus, an Hilbertian epsilon term is a constant representing this set and it cannot change its referent. In this sense it is a definite expression. This two-sided character of the epsilon is reflected in the sometimes contradictory application as illustrated in the following three sections.

### 3.2 Indefinite noun phrases and epsilon terms

For obvious reasons, epsilon terms are used to represent indefinite noun phrases. According to the first Hilbert rule in (16) each formula with an existential quantifier can be translated to a quantifier free formula with an epsilon term. Formula (20a) representing sentence (20), can be translated into the quantifier free formula in (20b).

A man comes.

(20a) $\exists x [\text{Man}(x) \& \text{Comes}(x)]$

(20b) $\text{Man}(\varepsilon x [\text{Man}(x) \& \text{Comes}(x)]) \& \text{Comes}(\varepsilon x [\text{Man}(x) \& \text{Comes}(x)])$

(20c) $\text{Comes}(\varepsilon x \text{Man}(x))$

The complexity of the epsilon term in (20b) is caused by the second part of epsilon
definition (19) according to which the choice function assigns an arbitrary element to the empty set. To prevent this possibility one has to guarantee that the chosen element of the set $\lambda x [\text{Man}(x) \& \text{Comes}(x)]$ is a man and comes, i.e. that set is not empty. The epsilon operator can replace the existential quantifier, but with the cost of very complex terms.

It was also proposed to use it as the semantic representation for the indefinite article. The epsilon term $\varepsilon x \text{Man}(x)$ representing the indefinite noun phrase *a man* is interpreted as the arbitrary choice out of the set of men. Since the choice is arbitrary, no definite object can be referred to. This view mirrors mathematical language games for proofs: “take an arbitrary number and ...”. This attractive approach was held by linguists who employ the epsilon operator as a counterpart of the iota operator for the indefinite article (e.g. Ballmer 1978). However, if the epsilon is not used as in (20b) but as in (20c) in order to represent (20) the whole reasoning becomes invalid. “Hilbert’s epsilon calculus is sometimes thought to formalise indefinite descriptions, like ‘an F’, ‘some F’, on account of the choice thus often available for the referent of ‘$\exists x \text{Fx}$: but this is a misaprehension” (Slater 1988a, 151). Slater proves this on basis of the following example:

(21) Some F are G and some F are H.
(21a) Some G are H.
(21b) G $\varepsilon x \text{Fx} \& H \varepsilon x \text{Fx}.$
(21c) $\exists x [\text{Gx} \& \text{Hx}]$

Sentence (21) does not entail (21a), but the representation in (21b) does entail (21c). The chosen representative of the set of F is an arbitrary element. But once determined, ever chosen. The two epsilon expressions in (21a) refer to the same object, whereas the two expression in (21) do not.

Summarizing this short discussion, epsilon terms do not stand directly for indefinite noun phrases. They are rather context dependent abbreviations for more complex terms.

**3.3 Definite noun phrases and epsilon terms**

Slater (1988b, 285) criticizes the use of the epsilon for indefinite noun phrases and proposes to use the epsilon for demonstrative noun phrases. “With a demonstrative expression we, first of all, have reference to one particular thing, as befits a constant term; but which particular thing is not, in general, specified linguistically within the term itself: it is, in general, merely the thing the speaker is pointing to, or the thing the speaker has in mind, so it is there, in the context, that indefiniteness comes in.” Contrary to this observation, Slater regards epsilon terms as constants, like proper names, and not as context dependent expression like indexicals or demonstratives. He does modify the referent of an epsilon term according to the intension of the descriptive material, but he does not make the second necessary step and vary the choice according to the context. He rather keeps the “rigidity” of the original epsilon calculus (Slater, this volume, section 4). Epsilon terms cannot change their reference during a text, i.e. all expressions *the F* are represented by the same epsilon term and refer, therefore, to the same object. This seems to be a reflex on the Russellian uniqueness condition, since the notions of uniqueness and the notion of rigidity are both static and global for a given text. However, it seems necessary to modify this conception of definite noun phrases. They are rather to be described in a dynamic
and more local approach, like indexicals. I propose a formal account of this context dependent epsilon analysis in sections 4 and 5.

### 3.4 E-type pronouns and epsilon terms

E-type pronouns are pronouns that are anaphorically linked to quantifier phrases such that the pronouns stand outside the syntactic scope of the quantifier. The representation of such pronouns with epsilon terms is the most obvious application of choice functions to natural language analysis. The character of E–type pronouns corresponds to the two-sided character of epsilon terms. On the one hand, E-type pronouns are regarded as referring expression like deictic pronouns. On the other hand, their antecedents are not referring terms but quantifier phrases that denote sets. The anaphorical relation can be seen in the conversion from a set to an object of this set. This conversion mirrors the definition of the choice function given above.

(22) A man comes. He whistles.

(22a) \( \exists x \ [\text{Man}(x) \& \text{Comes}(x)] \& \text{Whistles}(\varepsilon x \ [\text{Man}(x) \& \text{Comes}(x)]) \)

The representation (22a) of the first sentence of (22) expresses that the intersection of the set of men and the set of coming entities is not empty. The second part expresses that there is one arbitrary but fixed element of this intersection for which it holds that it whistles. The epsilon term reflects the function of the E-type pronoun that stands for a definite term that is a representative of a set. Since the epsilon does not claim any uniqueness condition the essential argument against the E-type analysis cannot be maintained. Therefore, choice functions are often used in E-type approaches that try to handle the problematic uniqueness condition of Russellian descriptions (Ballmer 1978, Slater 1988a,b, Chierchia 1992, van der Does 1993).

Still, there are two open questions in this approach. First, the transformational process that constructs the descriptive material of the epsilon term is not described. Second, in the static view assumed thus far each set gets exactly one representative. This assignment cannot be changed during a discourse, contrary to fact. Given the case that the two sentences in (22) are continued by a sequence of sentences, the last two of which are in (23). Then the choice function would necessarily select the same man as in (22a), although probably another man is meant. 4

(23) [...] A man comes. He does not whistles.

(23a) [...] \( \exists x \ [\text{Man}(x) \& \text{Comes}(x)] \& \sim \text{Whistles}(\varepsilon x \ [\text{Man}(x) \& \text{Comes}(x)]) \)

The use of the epsilon for representing indefinite and definite noun phrases or anaphorical pronouns overcomes the notoriously difficult uniqueness condition, but it raises the question of context dependency of these terms more urgently than the classical approach. In the next section I demonstrate that choice functions can only be used in linguistic theory if one makes them dependent on an additional context parameter.

### 4. Context dependent choice functions

As shown in section 3, all three applications of choice functions to linguistic categories are weakened by the fact that Hilbert’s epsilon is only defined for a whole
discourse. If a representative of a set is chosen once, it is used during the whole discourse for the named set and cannot be changed by further information. In all three discussed applications of the epsilon, it became obvious that a context dependent choice function is necessary, but only Egli (1991) proposed a formal treatment of this context dependency. Instead of one (static) choice function we assume a family of choice functions, each representing a different salience. In this way Lewis’ original idea of a salience hierarchy is reconstructed. Modified epsilon terms are employed to represent definite and indefinite noun phrases. The complex structure of epsilon terms allows us to capture functional definites and dependent expressions. Finally, I show that E-type pronouns can be best reconstructed as context dependent epsilon terms.

4.1 Salience and choice functions
The concept of salience was introduced into the discussion of the semantics of definite noun phrases in the 70’s (Lewis 1970, 1979; Kripke 1977; McCawley 1979) in order to replace Russell’s problematic uniqueness condition for definite descriptions: “The proper treatment of description must be more like this: ‘the F’ denotes x if and only if x is the most salient F in the domain of discourse, according to some contextually determined salience ranking” (Lewis 1979, 178). The notion of salience itself is influenced by the analysis of demonstrative expressions. A demonstrative like this man refers to the most prominent object in the physical environment of the speaker and hearer. Salience as used in the present approach, however, does not depend entirely on the physical circumstances, or any other single cause. Rather, it is a bundle of different linguistic and extra-linguistic factors, as already noted by Lewis (1970, 63): “An object may be prominent because it is nearby, or pointed at, or mentioned; but none of these is a necessary condition of contextual prominence. So perhaps we need a prominent-objects coordinate, a new contextual coordinate independent of the other.”

In the remainder of the discussion salience is treated as a property of a context. This property raises on object out of each set. It is the most salient or most prominent object of this set. An object a is the most salient object of the set F if a is the most prominent object among all objects of F in the context i. We also speak of the most salient F in the context i. The present approach treats salience as a primitive which will not be further analysed. The idea of salience has often been criticized because of its pragmatic nature (cf. Heim 1982), however, it has never been seriously attempted to give an explicit formal account of salience and Lewis’ “prominent-object coordinate”.

The salience hierarchy of a context i can be formally reconstructed by a context dependent choice function $\Phi_i$ that assigns one of its elements to each non-empty set. This context dependent choice function is determined by background knowledge, the direct physical situation or information that was given in the previous discourse. We can state the meaning of definite and indefinite noun phrases in terms of modified epsilon terms. Definite noun phrases refer to their referents according to the given salience ranking of the discourse. Indefinite noun phrases, on the other hand, do not refer to any salient object but to an arbitrarily chosen object. The definite noun phrase the F is represented as the context dependent epsilon term $\epsilon_i x \ F x$ and interpreted as
the current choice function $\Phi_i$ applied to the extension of $F$, i.e. the in $i$ selected $F$. Analogously, we represent the indefinite noun phrase $an$ $F$ as the modified eta term $\eta_x F x$. The eta term is interpreted by a choice function that differs from the current choice function at least in the element that is assigned to the set $F$. Thus it refers to an arbitrarily chosen element of the set $F$:

\[
(24) \quad \text{the F: } [[\varepsilon_i x F x]] = \Phi_i([[F]]) \quad \text{i given by the context}
\]

\[
(25) \quad \text{an F } [[\eta_i x F x]] = \Phi_k([[F]]) \quad \text{k new}
\]

The representation of definite and indefinite noun phrases as terms deviates from standard treatments that use quantifier phrases. However, already Hintikka (1974) argues that definite and indefinite noun phrases in natural language behave more as terms in predicate logic than like quantifier phrases. Following this line of reasoning, discourse representation theories (cf. Kamp 1981, Heim 1982) represent definite and indefinite noun phrases as free variables. The present view continues this tradition, but uses complex terms which can express dependencies between expressions, as shown in the next sections.

### 4.2 Definite noun phrases as modified epsilon terms

There are several different uses of definite noun phrases, some of which were already mentioned in section 3.3. Only the four most important uses of definite noun phrases will be considered: (i) the situational use of definite noun phrases as in (26), (ii) the use in cases of uniqueness as in (27), (iii) the functional use as in (28) and (29) and (iv) the anaphorical use as in (30).

\[
(26) \quad \text{The table is covered with books.}
\]

\[
(27) \quad \text{The first man in space was not an American.}
\]

\[
(28) \quad \text{The father of Bertrand Russell was bold.}
\]

\[
(29) \quad \text{Every man loves the woman that raised him.}
\]

\[
(30) \quad \text{A man comes. The man whistles}
\]

The logical form of each sentence (26)-(29) and its interpretation is discussed separately in this section. The anaphorical use of a definite noun phrase, as in (30), is analysed together with E-type pronouns in section 4.3. Sentence (26) receives the logical form (26a), which is interpreted in (26b) in a model $M$ and according to an assignment $g$. The atomic sentence is true if the extension of the argument lies in the extension of the predicate. The argument $\varepsilon_i x \ Table(x)$ gets its value by the interpretation rule (24) for definite noun phrases. It is the object that is selected by the choice function $\Phi_i$ that is determined by the context $i$.

\[
(26a) \quad \text{Covered_with_books}(\varepsilon_i x \ Table(x))
\]

\[
(26b) \quad [[\text{Covered_with_books}(\varepsilon_i x \ Table(x))]]^Mg = 1 \\
\quad \text{iff } [[\varepsilon_i x \ Table(x)]]^Mg \in [[\text{Covered_with_books}]]^Mg \\
\quad \text{iff } \Phi_i([[\text{Table}]]^Mg) \in [[\text{Covered_with_books}]]^Mg \\
\quad \text{iff the in the context i most salient table is covered with books}
\]

This analysis of situational dependent or incomplete definite noun phrases shows the basic mechanism of fixing the referent for a definite noun phrase by the principle of choice. Situational dependent definite noun phrases are the prototypical case for introducing this analysis, yet it also works for the other uses of definite noun phrases, as will be shown below.

In (27) the definiteness of the noun phrase is derived from the unique concept expressed in the appellative phrase *first man in space* which can only be fulfilled by
one object. In the interpretation of (27a) the context parameter of the choice function does not play any role since each choice function selects the same element out of a singleton. In contrast to the Russellian interpretation in (3), the definite noun phrase is given wide scope like in the case of the situational use or in the case of functional concepts.6

(27a) \[\neg \text{American}(\epsilon_{i}x \text{ First\_man\_in\_space}(x))\]
(27b) \[[\neg \text{American}(\epsilon_{i}x \text{ First\_man\_in\_space}(x))]^{M,g} = 1\]
    \[\text{iff } [[\text{American}(\epsilon_{i}x \text{ First\_man\_in\_space}(x))]^{M,g} = 0\]
    \[\text{iff } \Phi_{i}( [[\text{First\_man\_in\_space}]^{M,g}) \notin [[\text{American}]^{M,g}\]
    \[\text{iff the first man in space in context } i \text{ is not an American}\]

Functional expressions like (28) uniquely refer to an object because of their lexical content. The appellative phrase father of is assigned one value for each argument. For the argument Bertrand Russell it gets the set with the unique element that is the father of Bertrand Russell. Any choice function must assign to this set its unique element. Therefore, the interpretation does not depend on a particular context i.

(28a) \[\text{Bold}(\epsilon_{i}x \text{ Father\_of}(x, b))\]
(28b) \[[\text{Bold}(\epsilon_{i}x \text{ Father\_of}(x, b))]^{M,g} = 1\]
    \[\text{iff } \Phi_{i}(a | [[\text{Father\_of}(x, b)]^{M,g}/a = 1) \in [[\text{Bold}]^{M,g}\]
    \[\text{iff the in the context } i \text{ selected father of Bertrand Russell is bold}\]

Like the two earlier discussed cases, functional definite noun phrases receive wide scope. This correlates with the fact that the functional concept is of lexical origin. However, functional expressions can be made dependent on other parameters, as in (29). In the logical form (29a), the dependent definite description is represented as a complex epsilon term that contains a free variable. Thus the value of the whole term depends on the particular choice of this parameter. If this dependency is described in terms of scope, the epsilon term would have narrow scope in relation to the universal quantifier.

(29a) \[\forall x [\text{Man}(x) \rightarrow \text{Love}(x, \epsilon_{i}y [\text{Woman}(y) \& \text{Raised}(y, x)])]\]
(29b) \[[\forall x [\text{Man}(x) \rightarrow \text{Love}(x, \epsilon_{i}y [\text{Woman}(y) \& \text{Raised}(y, x)])]]^{M,g} = 1\]
    \[\text{iff for all } h = g^{x}/d \text{ with } [[\text{Man}(x))]^{M,h} = 1 \text{ it holds also}\]
    \[[[[\text{Love}(x, \epsilon_{i}y [\text{Woman}(y) \& \text{Raised}(y, x)])]]^{M,h} = 1\]
    \[\text{iff for all } h = g^{x}/d \text{ with } [[\text{Man}(x))]^{M,h} = 1 \text{ it holds also}\]
    \[<d, \Phi_{i}(a | [[\text{Woman}(y) \& \text{Raised}(y, x)])^{M,h} = 1)>) \in [[\text{Love}]^{M,h}\]
    \[\text{iff for every man it holds that he loves the in the context } i \text{ most salient woman, that raised him.}\]

To summarize this section, I showed that context dependent epsilon terms successfully represent definite descriptions. In the situational use the context dependency is of crucial importance for fixing the referent, whereas for definite noun phrases of unique or functional concepts the complex character of the epsilon term is essential. The context dependency does not play any role since the appellative phrase is fulfilled by exactly one object. A representation with epsilon terms preserves the grammatical functor-argument structure in the logical form and allows an interpretation of definite noun phrases in situ. In this sense, it is surface oriented.

4.3 E-type pronouns as modified epsilon terms

It was shown in the last section that intuitive and formal properties of definite noun
phrases can be captured adequately by context dependent choice functions. In this section, the analysis is extended to E-type pronouns, the second important manifestation of definite descriptions in linguistic theory. However, we will start the investigation with the anaphorical use of definite noun phrases. The anaphorical use of expression can be understood in terms of the indexical use applied to a discourse world. The definite noun phrase the table in (26) refers indexically to a salient object in the physical environment. The definite noun phrase the man in (30), however, refers to a discourse entity that was introduced by the previous discourse. The salience hierarchy according to which an anaphorical expression gets its referent is determined by the discourse information given so far. We reconstruct this fact by applying the choice function not to objects (of the “real” world) but to discourse referents. In the remainder of this section I discuss some of the intuitive advantages of this approach, followed in section 5 by the development of a formal fragment of this idea.

The anaphorical definite noun phrase the man in (30) is represented as the context dependent epsilon term \( \varepsilon^i_{\text{Man}}(x) \) which refers to the same object that was introduced by the eta term in the preceding sentence. The interpretation of (30) proceeds analogously to the examples (26-29) save that the choice function does not dependent on the extra-linguistic context, but rather on the discourse.

Sentence (30) is true according to the representation (30b) if the most salient man in an arbitrarily chosen situation \( k \) comes (with \( k \) differing from the given situation \( i \) at most in the assignment to the set of men) and if the most salient man in the context \( k \) whistles. The representation reflects the intuitive idea that an indefinite noun phrase not only refers to an arbitrarily chosen object, but also raises this object to salience and changes the current salience such that the subsequent definite noun phrase refers to this very object. Thus, the information on coreference is encoded into the salience hierarchy, which can be changed by linguistic items like indefinite noun phrases. The exact mechanism is described in section 5. Simplifying, we can reduce the principle of salience to the principle that an anaphorical expression refers to the last mentioned object in discourse. The salient discourse entity can also be picked up by a pronoun, which refers to the most salient object of all. The pronoun it in (31a) refers to the same object as the indefinite noun phrase a wine glass in the antecedent sentence. Therefore it can be replaced by the definite description the wine glass that broke last night as in (31b). Other than in the Russellian analysis no uniqueness condition is entailed. Thus, (31c) does not follow from the analysis and the uniqueness problem of E-type pronouns does not arise.

The same line of reasoning holds for the more complex donkey sentences. The pronoun it in (32) refers to the same object that was raised by the indefinite noun phrase a donkey. A further complication is that the universal quantifier, like conditionals, creates a hypothetical domain in which each assignment is tested. This
is generally expressed in the truth conditions for universal sentences. We cannot discuss this more general issue but concentrate rather on the analysis of the pronoun. The pronoun it in (32) can be paraphrased by the definite description the donkey that he owns in (32a) and represented by the complex epsilon term in (32b).

(32) Every farmer who owns a donkey beats it.
(32a) For every farmer, if the farmer owns the donkey, then he beats the donkey he owns.
(32b) $\forall x [(Farmer(x) \& Owns(x, \eta_i y Donkey(y)))$
\hspace{1cm} $\rightarrow Beats(x, \varepsilon_i y [Donkey(y) \& Owns(x, y)])]$

The claim is that the so-called definite reading (32b) is the only adequate representation for the donkey sentence. The general opposition between a strong and a weak reading is caused by a different treatment of the relevant choice function. An informal paraphrase for the weak reading (32c) is given in (32d), where the choice function is determined by the previous context and relates to each farmer only one donkey, let us say the most lazy one. The universal reading (32e), however, originates in an uncertainty about the criteria of choice. Therefore, we have to assume every possible choice as paraphrased in (32f). This explanation of the strong reading of donkey sentences has already been proposed by Gawron et al. (1991) and Chierchia (1992).

(32c) $\forall x [(Farmer(x) \& \exists y [Donkey(y) \& Owns(x, y)])$
\hspace{1cm} $\rightarrow \exists y [Donkey(y) \& Owns(x, y) \& Beats(x, y)]]$
(32d) There is a choice function $i$ such that for each farmer who owns a donkey the farmer beats the most salient donkey according to $i$.
(32e) $\forall x \forall y [(Farmer(x) \& Donkey(y) \& Owns(x, y)) \rightarrow Beats(x, y)]$
(32f) For all possible choice functions $i$: for each farmer who owns a donkey the farmer beat the most salient donkey according to $i$.

This intuitive consideration shows a way out of the donkey dilemma. The different readings of donkey pronouns and their antecedents do not originate in two underlying meanings of the indefinite noun phrase, but they are caused by the determination of the relevant choice functions.\(^8\)

5. Salience change semantics

As already noted in the last section, an anaphoric expression refers to a salient entity in the discourse which was introduced by another linguistic item. The only expression that can introduce a new entity and raise it to salience so far is an indefinite noun phrase. In the following sketch of a fragment of a salience change semantics, we distinguish between the denotational aspect of meaning and its salience change potential. In a preliminary system we agree on the following simplifications: only indefinite noun phrases modify the salience, there is a straightforward left to right interpretation of the terms, and we do not regard non-linguistic influences of a once given choice function.

The salience change potential of the indefinite noun phrase an $F$ changes a given salience by raising a new object out of the set of $F$ to salience regarding the set $F$. We further distinguish two types of expressions regarding their behavior relative to a modified salience: atomic sentences, conjunctions and sequences of sentences pass on a modified salience, whereas negation and conditionals block it. This distribution
corresponds to externally dynamic expressions in dynamic logics or discourse representation theories save for the atomic formula which is statically interpreted in these theories. For reasons of space, we discuss only the former group of (externally dynamic) expressions.

The salience change potential of an expression is represented at a separate level. Still, both aspects of meaning, the denotational aspect or the common interpretation and the salience change potential, interact with each other. When interpreting an expression, both levels must be considered. However, conceptual transparency calls for separate (formal) levels (cf. van der Does 1993). For the interpretation, we can keep to a classical predicate logic that is extended only by the epsilon operator and eta operator which are interpreted by the contextual parameter of salience in the form of a choice function. Expressions are interpreted in a model \( M = <D, I> \) with the individual domain \( D \) and the interpretation function \( I \) and according to an assignment \( g \) and a choice function \( \Phi \) which is different from the choice function used so far. Here, the choice function \( \Phi \) is not part of the model, but an additional parameter like the assignment. The salience change potential of linguistic expressions are computed separately. Each linguistic expression \( \alpha \) has a salience change potential, i.e. a function from choice functions into choice functions. We represent this function as \( \{\{\alpha}\}\). The meaning of an expression \( \alpha \) consists of its interpretation and its salience change potential:

(i) An expression \( \alpha \) is interpreted in a model \( M \) according to an assignment \( g \) and a choice function \( \Phi \): \( [\alpha]^{M,g,\Phi} \)

(ii) The salience change potential of an expression \( \alpha \) applied to the current choice function \( \Phi \) yields an updated choice function \( \Phi' \): \( \{\{\alpha\}\}(\Phi) = \Phi' \)

5.1 The dynamics of terms

As discussed earlier, indefinite noun phrases play the essential role in the salience change semantics. They do two different things at the same time. Firstly, they arbitrarily refer to an object or discourse referent which fulfills the descriptive content of the indefinite noun phrase. This referent must be new and not mentioned before. Secondly, they change the given salience hierarchy in such a way that the just chosen referent becomes the most salient one for the named set. Dynamic logic pays attention only to the first point and does not regard the salience shift potential of indefinite noun phrases. In the system of Groenendijk and Stokhof (1991) indefinite noun phrases are classically represented as existential quantifiers. However, the dynamic interpretation with non-deterministic programs is similar to the use of choice functions: “... in a dynamic set-up the interpretation of an indefinite description can be viewed as an act of picking an arbitrary individual, i.e. as an indeterministic action” (van Eijck 1993, 240; see also van Eijck and de Vries 1992). Therefore, van Eijck (1993) does not use the (dynamic) existential quantifier, but rather a dynamic eta term for representing indefinite noun phrases. However, it is not interpreted as a term creating operator, but rather as dynamic quantifier: “Note that \( \eta \) and \( \imath \) are program building operators (in fact, dynamic quantifiers) rather than term building operators, as in the logic of Hilbert & Bernays” (van Eijck 1993, 245). Furthermore, definite noun phrases are interpreted according to the Russellian uniqueness condition: “It is not difficult to see that this [i.e. the interpretation...
conditions for \( \iota \) results in the Russell treatment for definite descriptions.” (van Eijck, 1993, 245). But we have already seen that the uniqueness condition is too strong a condition for natural language descriptions (cf. section 2). It rather should be replaced by the more flexible principle of salience. 

We, therefore, give a general interpretation rule and a salience change rule for indefinite noun phrases. The indefinite noun phrase \( \text{an } F \) is represented as the \( \eta \)-term \( \eta x Fx \). We do not index the \( \eta \) by the salience hierarchy because it is not part of the object language. Instead the index for the salience hierarchy is part of the metalanguage where it reappears as the index on the choice function. The interpretation of the indefinite noun phrase \( \text{an } F \) is the object that is chosen by an arbitrary choice function \( \Phi[F/a] \). The salience change potential of an indefinite noun phrase applied to the given choice function \( \Phi \) yields the updated choice function \( \Phi[F/a] \). \( \Phi[F/a] \) stands for a choice function that differs at most in \( F \) from the current choice function \( \Phi \). \( \Phi[F/a] \) assigns the referent \( a \) to the set \( F \). For all other sets it selects the same representative as the original choice function \( \Phi \) as defined in (34):

\[
(33a) \quad [[\eta x Fx]]^{M,g,\Phi} = \Phi[F/a]([[F]]^{M,g,\Phi})
\]

\[
(33b) \quad [[\eta x Fx]](\Phi) = \Phi[F/a]
\]

\[
(34) \quad \Phi[F/a]([[G]]^{M,g,\Phi}) = \Phi([[G]]^{M,g,\Phi}) \text{ for all } G \neq F
\]

\[
\Phi[F/a]([[F]]^{M,g,\Phi}) = a
\]

The semantics of definite noun phrases is straightforward. Definite noun phrases refer to the most salient object with the expressed descriptive content according to the current salience hierarchy. We can state the general interpretation rule and the salience change rule for definite noun phrases. The definite noun phrase \( \text{the } F \) is represented as the epsilon term \( \varepsilon x Fx \). This term refers to the most salient \( F \) according to the given choice function \( \Phi \). As noted before, the given choice function may be determined by background knowledge, the direct (physical) environment and/or the linguistic context so far. A definite noun phrase does not change the given choice function \( \Phi \).

\[
(35a) \quad [[\varepsilon x Fx]]^{M,g,\Phi} = \Phi([[F]]^{M,g,\Phi})
\]

\[
(35b) \quad [[\varepsilon x Fx]](\Phi) = \Phi
\]

5.2 Dynamic atomic formulas

Having given the interpretation of terms, we can now develop the interpretation of atomic sentences. We first discuss the analysis of an atomic sentence with an indefinite noun phrase as an argument and then develop the general scheme. The interpretation of sentence (36a) is true iff the interpretation of the term lies in the extension of the predicate. This is the case iff there is a new choice function that differs from the current one in terms of the representative for the set of women. The interpretation (36b) is true under the same truth conditions as the discourse representation in (36c). The salience change potential of this atomic sentence is straightforwardly the potential of the indefinite term that is passed on, as expressed in (36d).

\[
(36a) \quad \text{A woman smiles.}
\]

\[
(36b) \quad [[\text{Smiles}(\eta x \text{Woman}(x))]^{M,g,\Phi} = 1
\]

\[
\iff [[\eta x \text{Woman}(x)]^{M,g,\Phi} \in [[\text{Smiles}]]^{M,g,\Phi}
\]

\[
\iff \text{there is a } \Phi[\text{Woman}/d] \text{ and }
\]

\[
\Phi[\text{Woman}/d]([[\text{Woman}]]^{M,g,\Phi}) \in [[\text{Smile}]]^{M,g,\Phi}
\]

17
We can now state the general interpretation rules for an atomic sentence in (37a). For each term we must consider one possible change in the salience hierarchy. An atomic formula with \( n \) terms becomes true if there is sequence of \( n \) choice functions such that each term is interpreted according to the corresponding choice function that was possibly modified by the interpretation of previous terms. Atomic formulas are not only internally dynamic but also externally dynamic. The updated salience that results from interpreting indefinite noun phrases is passed on to the following text. The salience change potential of an atomic sentence is computed in (37b) by application of the salience change potential of each term of the sentence to the previous salience.

\[
\begin{align*}
(37a) \quad [\text{[R}(t_1, \ldots, t_n)]^M,g,\Phi = 1 & \text{ iff there are } \Phi_1, \ldots, \Phi_n \text{ with } \Phi_1 = \Phi \text{ and } \\
([t_1])(\Phi_1) = \Phi_2 & \text{ and } ([t_2])(\Phi_2) = \Phi_3 \ldots \text{ and } ([t_{n-1}]) (\Phi_{n-1}) = \Phi_n \\
\text{ such that } <[t_1]^M,g,\Phi_1, [t_2]^M,g,\Phi_2, \ldots, [t_n]^M,g,\Phi_n> \in [[R]]^M,g,\Phi
\end{align*}
\]

\[
\begin{align*}
(37b) \quad [[G(t_1, \ldots, t_n)](\Phi) = ([t_n]) \ldots ([t_2]) ([t_1])(\Phi))
\end{align*}
\]

5.3 Conjunction and sequences of sentences

Sequences of sentences that are combined with a conjunction like \( \text{and}, \text{but} \) etc. and sequences that are not combined with any linguistic expressions are treated in the same way. The interpretation rule (38a) for conjunctions says that the conjunction is true if both conjuncts are true. The second conjunction is interpreted according to the choice function that is (possibly) updated by the first conjunct. This rule corresponds to the internal dynamics of conjunctions in other theories. The salience change potential (38b) of a conjunction results from the salience change of both conjuncts. This reflects the external dynamics of conjunctions.

\[
\begin{align*}
(38a) \quad [[\phi \& \psi]^M,g,\Phi = 1 & \text{ iff } [[\phi]]^M,g,\Phi = 1 \text{ and } [[\psi]]^M,g,([\phi])(\Phi) = 1 \\
(38b) \quad [[\phi \& \psi]](\Phi) = [[\psi]]([[\phi]](\Phi))
\end{align*}
\]

Generalizing these two rules to a text or a discourse, we can state (39a) for the interpretation of a text consisting of a sequence of sentences. Such a text is true if there is a sequence of choice functions, each of which results from the salience change potential of a sentence applied to the choice function of the preceding text, and if the sentences are true under the corresponding updated choice function.

\[
\begin{align*}
(39a) \quad [[[\psi_1 ; \psi_2 ; \ldots ; \psi_n]]^M,g,\Phi = 1 & \text{ iff there are } \Phi_1, \ldots, \Phi_n \text{ with } \Phi_1 = \Phi \text{ and } \\
([\psi_1])(\Phi_1) = \Phi_2 & \text{ and } ([\psi_2])(\Phi_2) = \Phi_3 \ldots \text{ and } ([\psi_{n-1}]) (\Phi_{n-1}) = \Phi_n \\
\text{ such that } [[\psi_1]]^M,g,\Phi_1 = 1 \text{ and } [[\psi_2]]^M,g,\Phi_2 = 1 \text{ and } \ldots \text{ and } [[[\psi_n]]]^M,g,\Phi_n = 1. \\
(39b) \quad [[[\psi_1 ; \psi_2 ; \ldots ; \psi_n]](\Phi) = [[[\psi_n]]] \ldots ([[[\psi_2]]] ([[[\psi_1]]](\Phi)])
\end{align*}
\]

With this general rule, we can analyse example (22-23), repeated as (40). This fragment of a text consists of series of sentences, two of which introduce a new discourse referent by the indefinite noun phrase \( \text{a man} \). The two definite noun phrases each refer to the last introduced referent. For most theories, this small text is difficult to interpret. According to the classical static view that represents definite descriptions with iota terms (see section 2.2), the representation (40a) cannot be interpreted as being true since the uniqueness condition is violated. The analysis with Hilbert’s epsilon in (40b) yields a contradictory result since it assigns to one and
the same epsilon term a property and its negation. But even a dynamic approach as
exemplified in (40c) has the problem of relating the anaphorical expression to its
antecedent. The point is that analyzing (40) we can see no reason for employing \(d_n\)
rather than \(d_1\) for the last occurrence of the definite noun phrase. The problem seems
to be rooted in the nature of discourse markers. Like variables, they are not anchored
to any overt items of natural language. In fact, this is only a new reincarnation of the
traditional problem of how to get the antecedent of an anaphorical term. The
traditional way to handle this is to interlock coreferring expressions by means of
coindexing as in (40d) - but this clearly does not solve the problem, but rather
only moves it outside the theory. It is assumed that the formal model starts where
coinDEXing, and hence anaphora resolution, is already done.

(40) A man comes. He whistles. [...] A man comes. He does not whistle.
(40a) \(\exists x \left[ Mx \land Cx \right] \land Wlx \left[ Mx \land Cx \right] \land \ldots \land \exists x \left[ Mx \land Cx \right] \land \neg Wlx \left[ Mx \land Cx \right] \)
(40b) \(\exists x \left[ Mx \land Cx \right] \land Wex \left[ Mx \land Cx \right] \land \ldots \land \exists x \left[ Mx \land Cx \right] \land \neg Wex \left[ Mx \land Cx \right] \)
(40c) \(\exists d_1, \ldots, d_n \mid M(d_1) \land C(d_1) \land W(d_1) \land \ldots \land M(d_n) \land C(d_n) \land \neg W(d_n) \}
(40d) \(\exists y \left[ My \land Cy \right] \land \ldots \land \exists y \left[ My \land Cy \land \neg Wy \right] \)

In the formal fragment proposed in this section, sentence (40) is
represented as (40e) with eta terms for the indefinite noun phrases and epsilon terms
for the definite noun phrases. The interpretation (40f) is decomposed according to the
interpretation rule for texts (39a). We have to assume for each sentence a choice
function that is built up by the preceding choice function modulo the salience change
potential of the last sentence: \(\Phi_n = \langle S_{n-1} \rangle(\Phi_{n-1})\). This mechanism stores the
information for fixing the referent of definite noun phrases, which is done in other
theories by coindexing. (40g) shows the result of various transformations. It becomes
obvious that the second definite noun phrase the man refers to a different referent
than the first occurrence of this expression. The second occurrence of the indefinite
noun phrase a man updates the choice function and this information is transmitted
to the last sentence where it gives the definite noun phrase its value. This
interpretation is true under the same conditions as the structure in (40h), but as
mentioned above, (40g) encodes the additional salience information that is necessary
for fixing the right referent for the second definite noun phrase.

(40e) \(C\eta x Mx \land W\exists x Mx \land \ldots \land C\eta x Mx \land \neg W\exists x Mx) \)
(40f) \([C\eta x Mx \land W\exists x Mx \land \ldots \land C\eta x Mx \land \neg W\exists x Mx]_M.g.\Phi_n = 1 \land \)
\([W\exists x Mx]\)\(\langle \Phi_2 \rangle = \Phi_3 \land \ldots \land [C\eta x Mx]\)\(\langle \Phi_{n-2} \rangle = \Phi_{n-1} \land \)
\([\neg W\exists x Mx]\)\(\langle \Phi_{n-1} \rangle = \Phi_n \) such that
\([C\eta x Mx]_M.g.\Phi_1 = 1 \land [W\exists x Mx]_M.g.\Phi_2 = 1 \land \ldots \land [C\eta x Mx]_M.g.\Phi_{n-2} = 1 \land [W\exists x Mx]_M.g.\Phi_{n-1} = 0. \)
(40g) \(\langle \Phi_2 = \Phi[M/d_1] \rangle \) and \(\Phi_{n-2} = \Phi[M/d_1][M/d_n] \) such that
\(\Phi[M/d_1][[M]_M.g.\Phi_1] \in [C]_M.g.\Phi_2 \land \Phi[M/d_1][[[M]_M.g.\Phi_2] \in [W]_M.g.\Phi_2 \)
\(\land \ldots \land \Phi[M/d_1][[[M]_M.g.\Phi_{n-2}] \in [C]_M.g.\Phi_{n-2} \land \)
\(\Phi[M/d_1][[M/d_n]_M.g.\Phi_{n-1}] \notin [W]_M.g.\Phi_{n-1} \)
(40h) \(\exists d_1 \ldots \exists d_n \left[ [M]_M.g. \land d_1 \in [C]_M.g. \land d_1 \in [W]_M.g. \land \ldots \land \right.
\(d_n \in [M]_M.g. \land d_n \in [C]_M.g. \land d_n \notin [W]_M.g. \)

This small and incomplete fragment of a salience change semantics was the final
illustration of the present theory that represents definite descriptions as context
dependent choice functions. The fragment needs to be extended in various directions. First, an extended salience change potential for definite and indefinite noun phrases must be worked out. Both expressions raise not only an element of the named set to salience but also elements of the supersets in order to license anaphorical relations between pronouns and their antecedents. Second, the semantics of negation and conditionals must be considered before a successful attempt at a formal analysis of donkey sentences can be undertaken.

6. Conclusion
In this paper I have discussed three theories of definite descriptions: Russell’s classical approach, Hilbert’s epsilon theory and the modified epsilon view. All three theories were tested as to whether they could adequately analyse the two manifestations of definite descriptions in linguistic theory, namely definite noun phrases and E-type pronouns. Russell’s classical theory fixes the reference of a definite description exclusively by descriptive material and the uniqueness assumption. His static view represents the standard analysis of definite noun phrases in semantics. However, a Russelian analysis of E-type pronouns is controversial. As an alternative to Russell’s theory, Hilbert’s epsilon is generally employed. Epsilon terms refer to exactly one object due to the descriptive content of the definite description and the principle of choice. Thus the problematic uniqueness condition does not arise. At first sight, epsilon terms seem to be the adequate representation of E-type pronouns since they select one object out of a given set. The application to definite and indefinite noun phrases is rather controversial. A closer investigation into the nature of Hilbert’s epsilon reveals that Hilbert’s static view must be elaborated. Only a dynamic approach with modified epsilon terms is able to represent both manifestations of definite descriptions: definite noun phrases and pronouns are represented as context dependent epsilon terms, whose referents are determined by the descriptive material, the principle of choice and the contextual salience hierarchy. They are interpreted by choice functions that depend on the context, which is constituted either by world knowledge or by the information of the preceding discourse. This approach not only analyses the different uses of definite noun phrases successfully, but it also shows a feasible treatment of E-type pronouns. Thus, the concept of definite descriptions can be represented uniformly as context dependent choice functions.

Notes
* The research for this paper was supported by the German Science Foundation for the project “Interaktion von Lexikon und Semantik”. Further, I should like to express my thanks to B. H. Slater and S. Akama for inspiring comments and M. Butt and J. Cole for helping me with the English. All remaining faults go to my account.
1. For a comprehensive discussion of these two positions, see Heim (1991).
2. For a more detailed critic see von Heusinger (1994, 382).
3. Extensionality is too strong a condition for natural language descriptions, as Slater (this volume, section 2) convincingly proves. However, the aim of the present paper is to show the situational dependency of epsilon terms. And this dependency cannot be captured by the notion of intensionality, as I argue in section 3.4 below.
4. Examples (22) and (23) are criticized as not being very natural, since one would
expect another man instead of a man in (23). This is true if the sentences in (23) stand sufficiently close to those in (22). However, if one assumes some context in between, the argument looses its value. This become obvious if we modify Slater’s example (this volume, section 4) in the following way:

(i) We have a dog and a cat and I think we are going to have to keep them under better control. They seem to work badly together defending the territory around our house. Yesterday a dog jumped over the wall into our front yard, where the cat was hunting a mouse. The dog chased the cat and bit it.

The last expression the dog refers to the last introduced dog and not to the first introduced one. Here, I agree with Lewis (1979) who assumes possible shifts in the salience hierarchy that determines the choice of the selected object. This view is basically dynamic and local, contrary to Slater’s global and static approach.

5. For a more formal modeltheoretic interpretation of context dependent epsilon terms, we have to extend our individual domain \( D \) of the model \( M \) by the set of indices \( \mathbb{I} \). Further we have to add the function \( \Phi \) that assigns to each index \( i \) of \( \mathbb{I} \) a choice function \( \Phi_i \). The interpretation of an epsilon term \( \varepsilon_i x Fx \) is as follows:

(i) \[ [[\varepsilon_i x \alpha]]^{M,g} = \Phi([i])(\{a: [[\alpha]]^{M,g}_{a/x} = 1\}) \]

(ii) \[ [[\eta_i x \alpha]]^{M,g} = \Phi([k])(\{a: [[\alpha]]^{M,g}_{a/x} = 1\}) \]

with \( k \) such that \( \Phi([k])(\{a: [[\alpha]]^{M,g}_{a/x} = 1\}) \neq \Phi([i])(\{a: [[\alpha]]^{M,g}_{a/x} = 1\}) \)

The index of the eta term expresses the context dependency of the indefinite noun phrase an \( F \), since the context parameter has to consider the linguistic context, as well. The sentence A man comes can be represented as \( \text{Come}(\eta_i x \text{Man}(x)) \), which is equivalent to (20b) if one assumes that there is at least one man. For there is at least one choice function such that the selected man comes (see von Heusinger 1992).

6. A modified epsilon term representing a definite noun phrase can get narrow scope regarding other operators either by containing a variable that is bound by that operator, as it was demonstrated in (29a), or by modality operators that capture the context index of the epsilon (see von Heusinger 1992).

7. Of course, this view is very simple. From the research in anaphora resolution, we know that many more factors are intertwined in anaphorical relations.

8. For a more detailed analysis see von Heusinger (1992, chapter 5).

9. There are two ways to include the necessary restrictions on the choice of an adequate referent for the indefinite noun phrase. Either one can define the eta term as a program in dynamic logic. The program collects all possible choices of the referent and then tests the results against the subsequent text (cf. van Eijck 1993). Or one assumes that there is at least one adequate change in the salience such that the newly chosen object fulfills the predicates in the following text. This move is more in the spirit of DRT.

References


Heim, Irene 1990. E-Type Pronouns and Donkey Anaphora. Linguistics and Philosophy 13, 137-177.


