THE REFERENCE OF INDEFINITES*

The indefinite article has the function of picking out a single representative from among various representatives of a kind.¹

1 INTRODUCTION

In this paper I argue that indefinite NPs have a more complex referential nature than is usually supposed, and that this structure must be reflected in their semantic representation. According to the classical view due to Frege and Russell, an indefinite NP is represented by an existential quantifier, a variable, the restriction and the occurrence of the variable in the argument position of the main predicate. Hence, there is no clear correspondence to the indefinite NP on the surface. Sentence (1) is translated into the formula (2a), in which the indefinite NP *a man* corresponds to the variable *x* in the argument position of the predicate *walk* and in the predication *man(x)*. The formula specifies that the intersection of the two sets denoted by the predicates is non-empty. The model-theoretic interpretation (2b) links the variable *x* to an object *d* that fulfills both predicates, treating the attributive material *man* on par with the assertive material *walk*. Hence, at the representational level, the indefinite NP is not represented as an independent expression. This conception has been widely accepted in semantics and can be found in current semantic theories.

(1) A man walks
(2) a. $\exists x [\text{man}(x) \& \text{walk}(x)]$
   b. The formula $\exists x [\text{man}(x) \& \text{walk}(x)]$ is true iff there is an object *d* in the domain of individuals such that *d* is in the extension of the predicate *man* and in the extension of the predicate *walk*.

In Lewis-Heim-Kamp theories, indefinites do not express existential force by their own; they rather introduce discourse referents into an additional level of semantic representation. The discourse referents can then be bound by other quantifiers or by the text operator $\exists$, as in (3a). Alternatively, we can describe the existential closure at the level of interpretation, as in (3b): the representation becomes true if there is an assignment functions that fulfills the conditions.

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As in the classical theory, this approach treats the descriptive material of the indefinite like the assertive material of the matrix sentence, and there is no clear correspondence between the syntactic constituent of an indefinite NP and its representation.

I want to maintain that indefinite NPs must have a different representation, which reflects their syntactic nature as proper constituents and their semantic function as referring expressions. I shall argue that indexed epsilon terms give a far better analysis of indefinites than the representation as existential quantifiers, as variables, or as discourse referents can do. Sentence (1) is represented by the formula (4), in which the epsilon term $\varepsilon x \text{man}(x)$ corresponds to the grammatical constituent $a \text{man}$. The epsilon operator is interpreted by a choice function $\Phi_n$, which assigns to a set one of its elements. This semantics reflects the primarily referential nature of indefinites. The model theoretic interpretation (5) is true if there is a choice function $\Phi_n$ such that the choice function assigns an element to the set of men that is in the extension of walking entities.

(4) \[ \text{walk}(\varepsilon x \text{man}(x)) \]

(5) The formula $\text{walk}(\varepsilon x \text{man}(x))$ is true iff there is a choice function $\Phi_n$ such that $\Phi_n([\text{man}]) \in [\text{walk}]$

I argue that this representation allows to analyze the complex structure of indefinite NPs, which is reflected in its interaction with quantifiers and adverbs of quantification. Furthermore, I argue that indefinites can also be dependent on other indefinite NPs, and I give a representation of this dependency structure for the first time.

The paper is organized as follows: Section 2 gives a short overview over the different analyses of indefinite NPs through history, starting from the traditional grammatical view, passing through the Fregean logic and ending with the dynamic account. Section 3 present recent theories that analyze indefinites by means of choice functions. The discussion of some problems of this approach leads to the modification proposed in the dynamic semantics with choice functions in section 4. Besides their interpretation as choice functions, indefinites also introduce updates on a global choice function in order to model their context change potential. It is only in this semantics that we can account for the uniform analysis of indefinites and definite NPs as terms. Furthermore, we can analyze dependencies between indefinite NPs and account for the so called asymmetric readings of conditionals.

2 INDEFINITE NPS AND THEIR REPRESENTATION

In this section I give a short overview of different approaches to and representations of indefinite NPs. Since the treatment of indefinite NPs cannot be separated from anaphoric expressions that are linked to them, this overview must encompass anaphora, too.
2.1 The traditional grammarian view

Traditional grammarians regard indefinite NPs, like an old man ( . . . ) in (6), as ‘individualizing’ and, therefore, as referential expressions, similar to definite NPs, proper names, and demonstratives: An indefinite NP refers to a physical (or fictional) object. Subsequent anaphoric expressions can denote the same object establishing anaphoric reference.

(6) An old man with steel rimmed spectacles and very dusty clothes sat by the side of the road. ( . . . ) He was too tired to go any further.²

In this referential view, indefinite NPs behave like expressions without scope: they are interpreted independently from other expressions, and they do not influence other scope sensitive expressions. Other uses of indefinites are derived from the referential one. Anaphoric pronouns can be understood as expressions that ‘stand for’ or ‘go proxy’ for the antecedent and, therefore, refer to the same object. The direct referential character of indefinites has been criticized since Frege, who, however, dismissed any referential aspect of indefinites.³

2.2 The classical view and scope relations

Frege was too concerned with ontological and epistemological considerations to realize the grammatical nature of indefinites illustrated above. In ‘Über Begriff und Gegenstand’ (1892) [“On Concept and Object”], he accounts for the difference between a ‘concept’ (‘Begriff’) and an instantiation of such a concept, i.e. an ‘object’ (‘Gegenstand’). He then correlates both with the grammatical terms ‘predicate’ and ‘argument’.⁴

The concept (as I understand the word) is predicative. On the other hand, a name of an object, a proper name, is quite incapable of being used as a grammatical predicate.

He concludes that the indefinite article marks a name for a predicate or for a concept, whereas the definite article indicates a name for an object.⁵

This is in full accord with the criterion I gave – that the singular definite article always indicates an object, whereas the indefinite article accompanies a concept-word.

Frege’s distinction between concept and object was codified in his representation of indefinites as existential quantifiers and definites as singular terms splitting the grammatical category of NP into two semantic categories. Russell later assimilated the representation of definite NPs to that of indefinites, i.e. as quantifiers. Owing to Montague and others, this representation became the standard, or classical, interpretation of definite and indefinite NPs in formal semantics.

Frege and Russell further noted that certain occurrences of indefinite NPs exhibit a dependency structure which is quite similar to the scope sensitive behavior of the existential quantifier in predicate logic. Their observations concern existential sentences like (7), negation like (8) and ambiguous sentences like (9). The indefinite NP
in the existential sentence (7a) can be quite appropriately represented by the quantifier in (7b). The representation is true if (7c) holds, i.e. if there is an object that is a pontoon. In (8b), the negation gets wider scope than the existential quantifier expressing that there is no object corresponding to the indefinite NP in (8a). And finally, the two intuitively available readings of (9a) differ in the dependency of the indefinite NP on the universal expression. In predicate logic, the two readings are represented by a different order of the operators involved, which determines their interpretation.

(7) a. There was a pontoon bridge across the river.
   b. \( \exists x [\text{pontoon}(x) \land \ldots] \)
   c. There is a \( d \) in the domain of individuals such that \( d \) is a pontoon...

(8) a. There was no cast net.
   b. \( \neg \exists x [\text{cast net}(x)] \)
   c. There is no \( d \) in the domain of individuals such that \( d \) is a cast net.

(9) a. Every fisherman is subsidized by a state organization.
   b. \( \forall x [\text{fisherman}(x) \rightarrow \exists y [\text{state organization}(y) \land \text{subsidize}(y,x)]] \)
   c. For all \( d \) in the domain: if \( d \) is a fisherman than there is an \( e \) in the domain such that \( e \) is a state organization and \( e \) subsidizes \( d \).
   d. \( \exists y [\text{state organization}(y) \land \forall x [\text{fisherman}(x) \rightarrow \text{subsidize}(y,x)]] \)
   e. There is an \( e \) in the domain such that \( e \) is a state organization and for all \( d \) in the domain: if \( d \) is a fisherman than \( e \) subsidizes \( d \).

2.3 The lexical ambiguity theory

Fodor & Sag (1982) propose a lexical ambiguity of the indefinite article giving up a uniform analysis of indefinites. Indefinites have either a specific or referential reading, as assumed by the traditional grammarians, or they have a non-specific or existential reading, as in the Fregean analysis. Fodor & Sag assume that the contrast between the two readings is incommensurable. They illustrate this point by the interaction of indefinites with quantifiers and definite NPs, as in (10a). The indefinite has either a specific reading or a non-specific reading. The classical approach to this contrast is by means of different scope: the indefinite NP can get wide or narrow scope with respect to the definite NP the rumor, reflecting the specific and non-specific reading, respectively. However, the universal phrase each student in (11a) cannot receive wide scope, as in (11c), due to an island constraint.

(10) a. John overheard the rumor that a student of mine had been called before the dean.
   b. the rumor ... there is a student ...
   c. a certain student ... the rumor ... he ...

(11) a. John overheard the rumor that each student of mine had been called before the dean.
   b. the rumor ... each student
   c. *each student ... the rumor
This means that the indefinite is either represented by an existential quantifier with different properties from other quantifiers, such as the possibility to move out of islands, or the indefinite NP is represented by different means. Fodor & Sag choose the latter view and propose that the indefinite NP is either interpreted as an existential quantifier or as a referring expression. The quantificational interpretation, as in (10b), must observe island constraint like other quantifiers and accounts here for the non-specific reading. The referring expression is scopeless like proper names and demonstrative, i.e. it behaves as if it always had widest scope, as in (10c).

This theory makes a clear prediction: an indefinite is interpreted either as a referential term and receives always widest scope, or as an existential quantifier, which has to obey scope islands. We can now test this prediction on examples with two quantifiers as in (12) or (13). In both sentences, there are two quantifiers beside the indefinite, which stands in a scope island. According to Fodor & Sag’s theory, we would only expect a narrow scope reading by the existential interpretation and a wide scope reading by the referential interpretation, but no intermediate reading. While judgements on intermediate readings are quite intricate, Farkas (1981) observed on examples, like (13), that intermediate readings are often very natural. (13) has a reading according to which for each student there is one condition such that the student comes up with three arguments against the condition.

(12) Each teacher overheard the rumor that a student of mine had been called before the dean.

(13) Each student has to come up with three arguments that show that some condition proposed by Chomsky is wrong.
   each student ... some condition ... three arguments ...

2.4 Discourse referents and dynamic binding

The classical view represents indefinite NPs as existential quantifiers that are scope sensitive in order to explain certain readings and ambiguities. Anaphoric pronouns are reconstructed by bound variables, which seems to be an adequate analysis up to the sentence level. However, Geach (1962) among others has shown that this does not work properly across sentences. In (14b), the last occurrence of the variable \( x \) cannot be bound by the existential quantifier since it is outside of the quantifier’s scope. The same holds for the conditional (15a), where the last occurrence of the variable \( y \) in the consequence of (15b) cannot be bound by the existential quantifier that is subordinated in the antecedent. Another problem is that in the intuitive reading (15c) of sentence (15a) the indefinite NP gets universal force, while the indefinite in (14a) has existential force.

(14) a. A fisherman walks. He whistles.
   b. \( \exists x [\text{fisherman}(x) & \text{walk}(x)] & \text{whistle}(x) \)

(15) a. If a fisherman catches a fish he sells it.
   b. \( \forall x [\text{fisherman}(x) & \exists y [\text{fish}(y) & \text{catch}(x, y)] \rightarrow \text{sell}(x, y)] \)
   c. \( \forall x \forall y [\text{fisherman}(x) & \text{fish}(y) & \text{catch}(x, y)] \rightarrow \text{sell}(x, y)] \)
Two approaches have been developed to solve these puzzles: One approach represents anaphoric pronouns as E-type pronouns, i.e. as complex terms like definite descriptions (cf. Evans 1977, Neale 1990). The other approach introduces a more flexible concept of ‘semantic scope’ that allows ‘dynamic’ binding beyond the syntactic scope of classical predicate logic. In this view, indefinites introduce a variable or a discourse referent and an open sentence associated with it (cf. Kamp 1981, Heim 1982). The variables can be bound by various operators, such as adverbs of quantification (see below), existential closure operations as in (14c) or conditionals as in (15d). The existential text-closure binds all free variables that are not yet bound by other operators. The conditional is represented as an unselectively binding universal operator yielding the classical and so called strong reading of a donkey sentence. The sentence is intuitively true if it holds for every fisherman that he sells each fish he has caught:

(14)  c. \( \exists \{x \mid \text{fisherman}(x) & \text{walk}(x) & \text{whistle}(x) \} \)  

(15)  d. \( \forall \{\langle x,y \rangle \mid \text{fisherman}(x) & \text{fish}(y) & \text{catch}(x,y) \}, \{\langle x,y \rangle \mid \text{sell}(x,y) \} \)  

This analysis is too coarse-grained as illustrated by (16). The unselective binder MOST that translates usually binds all cases, i.e. both variables, yielding the logical form (16b). However, this representation does not reflect the intuitive truth conditions of (16a), but gives rise to the well known proportion paradox (Bäuerle & Egli 1985, Kadmon 1987). The representation (16b) counter-intuitively becomes true in a situation where 99 farmers have one donkey each and they do not beat their unique donkey and where one farmer beats all of his 100 donkeys. Intuitively, one has to count donkey-owning farmers as in (16c), rather than farmer-donkey pairs. The standard solution to this problem is an additional existential closure rule (Kadmon 1987, Chierchia 1992) that binds one variable. However, this approach must then explain how the anaphoric pronoun it in (16a) can be bound. This is generally done by an accommodation rule which copies descriptive material from the antecedent clause into the clause with the anaphoric pronoun, as in (16d). Still (16d) does not reflect the intuition that the farmer beats the donkey he owns.

(16)  a. If a farmer owns a donkey he usually beats it.  

b. MOST(\( \{\langle x,y \rangle \mid \text{fisherman}(x) & \text{donkey}(y) & \text{own}(x,y) \} \), \( \{\langle x,y \rangle \mid \text{beat}(x,y) \} \)  

c. Most donkey-owning farmers beat a donkey they own.  

d. MOST(\( \{x \mid \text{fisherman}(x) & \exists y \text{donkey}(y) & \text{own}(x,y) \} \), \( \{x \mid \exists y \text{donkey}(y) & \text{own}(x,y) & \text{beat}(x,y) \} \)  

Summarizing the discussion in this section, a semantics of indefinites has to account for the following three points:

- Indefinites cannot (always) be represented as quantifiers, since they do not obey scope islands. They rather exhibit a great flexibility in their dependency on another operators.
• Indefinites depend in their interpretation on the context; but they also contribute to the context, in order to license anaphoric relations. In dynamic semantics, this contribution to the context is reconstructed by some update operation.

• Adverbs of quantification show more readings than it is expected from a representation of indefinite NPs by variables. In order to account for asymmetric readings, we have to assume a more fine-grained dependency structure between indefinite NPs.

3 Choice functions and the semantics of indefinites

Choice functions have recently become a fashionable tool for representing indefinites (cf. Reinhart 1992, Kratzer 1998, and the contributions in this volume of Peregrin, Slater, von Stechow, and Winter). Before I propose a solution to the dependency structure of indefinites, I present semantic approaches that use choice functions or their syntactic equivalent, epsilon terms, to represent indefinite NPs. Hilbert & Bernays (1939) were the first who defined the epsilon operator, which they used for metamathematical inferences. This classical formalism must be extended in at least two directions in order to be applicable to linguistic problems. First, we must assume a family of choice functions instead of one choice function given by the model, and second we have to embed this into a dynamic framework. In this section, I explore the first extension, while the dynamic framework will be developed in section 4.

3.1 The classical epsilon calculus

Hilbert & Bernays replaced the existential and universal quantifiers by epsilon terms for metamathematical reasons. They use the epsilon operator as a generalized iota operator without the uniqueness and the existential condition. The epsilon operator is used to replace the existential and universal quantifier, according to the two epsilon rules (17) and (18). The latter one can be inferred from the former by substitution of \( \neg F \) for \( F \), contraposition and replacement of the existential quantifier by the universal in (18).

(17)  
\[
\exists x F x \equiv F(\varepsilon x F x) \\
\exists x \neg F x \equiv \neg F(\varepsilon x \neg F x) \quad \text{[substitution of } \neg F \text{ for } F] \\
\neg \exists x \neg F x \equiv \neg \neg F(\varepsilon x \neg F x) \quad \text{[contraposition]}
\]

(18)  
\[
\forall x F x \equiv F(\varepsilon \neg F x) \quad \text{[replacement of the existential quantifier]}
\]

According to these syntactical definitions for the epsilon operator, the following interpretation becomes most natural: An epsilon term \( \varepsilon x F x \) is interpreted in a model \( M \), consisting of an individual domain \( D \), an interpretation function \( I \), and a choice function \( \Phi \), as that individual that is assigned to a set \( F \) by the choice function \( \Phi \). A choice function is generally defined as a function that assigns to each non-empty set \( s \) one of its elements, and an arbitrarily chosen element to the empty set.

(19)  
\[
[\varepsilon x F x]_{M,\Phi} = \Phi([F]_{M,\Phi}), \text{ with } \Phi \text{ as a function given by } M = \langle D, I, \Phi \rangle
\]

(20)  
\[
\Phi(s) \in s \text{ if } s \neq \emptyset \text{ and } \Phi(s) \in D \text{ if } s = \emptyset
\]
In this way, the universal and existential quantifiers can be replaced. In the following, we will concentrate on the representation of indefinite NPs. We argue that an epsilon term is more appropriate to mirror the nature of indefinite NPs than an existential quantifier. This was already noted by Hintikka (1976, 209f):

There exists one particularly natural way of looking at quantifiers which has never been put to use entirely satisfactorily before. It is to consider quantifiers as singular terms. It is plain even to a linguistically naked eye that quantifier phrases like ‘some man’, ‘every woman’, ‘a girl’, and even phrases like ‘some boy who loves every girl’ behave in many respects in the same way as terms denoting or referring to particular individuals. In view of such obvious facts, it seems eminently desirable to try to treat quantifier phrases both syntactically and semantically in the same way as singular terms.

3.2 The indexed epsilon calculus

Since Hilbert applied his epsilon terms only to the domain of numbers, a naturally ordered set, no determined choice function was necessary. However, in natural language the objects we refer to are not naturally ordered; rather, the order depends on a particular context. Thus, most attempts to introduce the epsilon operator into linguistic analysis have failed since they did not consider this context dependency. Egli (1991) approached this problem by assuming a family of choice functions for representing definite NPs and indefinite NPs. Each context $c$ has its own choice function $\Phi_c$, such that the definite NP $\text{the } F$ can be represented as the indexed epsilon term $\varepsilon_{c,x}Fx$, which can be paraphrased with the selected $x$ in the context $c$ such that $x$ is $F$ or the most salient $x$ in $c$ such that $x$ is $F$. It is interpreted as the element that results from applying the choice function $\Phi_c$ to the set of all $Fs$. The contribution of the context to the interpretation of the definite NP consists in an ordering of the elements of each set described in that context. In this – preliminary view – definite NPs are interpreted similarly to deictic expressions. The “unique availability” of the referent (cf. Peregrin, this volume) is warranted by the definition of the choice function, which assigns one element to a set.

Egli (1991) and von Heusinger (1997a) have generalized this semantics of definite NPs to indefinite NPs. Indefinite NPs are also represented by indexed epsilon terms, but here the index is not determined by the context, but free.

\begin{align*}
\text{the } F: \& \varepsilon_{c,x}Fx = \Phi_c(\llbracket F \rrbracket) & \text{ with } c \text{ contextually determined} \\
\text{an } F: \& \varepsilon_{i,x}Fx = \Phi_i(\llbracket F \rrbracket) & \text{ with } i \text{ free}
\end{align*}

Like free variables for individuals in Lewis-Heim-Kamp theories, the free index of the epsilon operator can be bound by operators in its environment or it can be existentially closed by some existential text operator. Thus, the contrast between definite and indefinite NPs roughly corresponds to the familiarity condition of Heim (1982). The advantages of using choice function variables instead of individual variables are the following: (i) the epsilon term corresponds to the syntactic constituent of a definite or indefinite NP, and the descriptive material of the indefinite is not treated on par with
main predicate in the sentence. Thus we can distinguish the identification of the referent from the assertion in the sentence. (ii) indefinites need not be moved or raised for expressing different dependency behaviors. They remain *in situ*, whereas the choice function variable can be bound by other operators. This explains different readings of the indefinite, as it will be shown in the next subsection. (iii) the assumption of free choice function variables squares with the theory of free indices of Farkas (this volume). (vi) this view clears the way for a dynamic semantics, in which the contextual change potential is expressed in updating choice functions, as it will developed in section 4.

### 3.3 Logical form with choice functions

The epsilon term $\varepsilon x \text{lion}(x)$ standing for *a lion* is interpreted as the operation of picking one element out of the set of lions. In the absence of any operator we assume an existential closure over epsilon indices or choice functions at the sentence level. Thus, the indefinite NP *a lion* refers to an arbitrarily chosen lion. The classical theory represents indefinite NPs as existential quantifiers in (23b). Discourse representation theories free the quantificational force from the representation of the indefinite. Indefinites are represented as free variables in (23c) that are associated with predicates (conditions) by the interpretation rules. The epsilon approach represents indefinite NPs in (23d) as indexed epsilon terms reflecting the argument structure of a sentence in a quite natural way. In the absence of any other operator, the indices are bound by the existential text closure $\exists$, which is interpreted as (23e).

\[(23)\]

a. A lion ate a zebra.

b. $\exists x \exists y [\text{lion}(x) \& \text{zebra}(y) \& \text{eat}(x, y)]$

c. $\left\{ (x, y) \mid \text{lion}(x) \& \text{zebra}(y) \& \text{eat}(x, y) \right\}$

d. $\exists \text{eat}(\varepsilon x \text{lion}(x), \varepsilon y \text{zebra}(y))$

e. (23d) is true iff there are choice functions $\Phi_1$ and $\Phi_2$ such that

$\langle \Phi_1([\text{lion}]), \Phi_2([\text{zebra}]) \rangle \in [\text{eat}]$

A very similar approach was developed by Reinhart (1992), Winter (1997), Kratzer (1998) among others. Influenced by the use of choice function for *wh*-phrases in islands of Engdahl (1986), it is assumed that indefinites in islands are represented by choice functions. In these approaches, sentence (23a) is represented as (23f), where the choice functions are represented by the variables $f_1$ and $f_2$, and the condition $CH(f_i)$, asserting that this function is a choice function. This representation is similar to the use of Skolem function in formal semantics (cf. von Stechow (this volume), and Winter 1997).

\[(23)\]  

f. $\exists f_1, f_2 [CH(f_1) \& CH(f_2) \& \text{eat}(f_1(\text{lion}), f_2(\text{zebra}))]$

Nevertheless, I maintain the indexed epsilon representation because the syntactic function of the article, namely its term creating force, is encoded in the syntax. Only the particular choice of which element is assigned to a set is variable and described as dependent on the semantic environment. Note that the formulae (23b) and (23c) are not
equivalent to the formulae (23d) and (23e) if there is no object that fits the descriptive material of the indefinite NP. However, in the remainder of the paper we will assume that there are always objects that fit the descriptive content.\footnote{9}

3.4 Dependent indefinite NPs

The choice of a particular referent of an indefinite NP can depend on the linguistic environment the indefinite is located in. This is generally illustrated by the interaction of a universal quantifier phrase like every lion and an indefinite NP like a zebra in (24a). Here the choice of the referent for the indefinite NP can depend on the particular choice for the lion, in which case the choice of the referent for the indefinite co-varies with the choice of the particular referent for every lion. This reading is classically represented by the formula (24b), where the universal quantifier precedes the existential one for the indefinite. The reading in which the referent of the indefinite is chosen independently of the particular choice for the universal quantifier is classically represented as (24c) with wide scope of the existential quantifier.

(24)  
\begin{enumerate}
\item Every lion ate a zebra.
\item $\forall x[\text{lion}(x) \rightarrow \exists y[\text{zebra}(y) \land \text{eat}(x,y)]]$
\item $\exists y[\text{zebra}(y) \land \forall x[\text{lion}(x) \rightarrow \text{eat}(x,y)]]$
\end{enumerate}

The dependent reading can also be represented by means of Skolem functions as in (25), which is equivalent to (24b). Here the Skolem function assigns to each lion a zebra that the lion ate. Skolem functions express the dependency of the value of one term from the value of another term in a more transparent way. They are the prominent means to represent dependent E-type pronouns, which are sometimes called “paycheque-pronouns” (cf. Karttunen 1969, 114). In (26a) the choice of the referent for the pronoun it depends on the value for the subject. This is represented by the Skolem function $f$ in (26b) that assigns a paycheque to each individual.\footnote{10}

(25) \quad $\forall x[\text{lion}(x) \rightarrow \text{eat}(x,f(x))]$

\hspace{1cm} \textit{f}: \text{Skolem function from lions into zebras they ate}

(26)  
\begin{enumerate}
\item Every man except John put his paycheque in the bank. John gave it to his mistress.
\item Every man except John put his paycheque in the bank. John gave $f(\text{John})$ to his mistress.
\end{enumerate}
\hspace{1cm} \textit{f}: \text{a Skolem function from individuals into their paycheques}

Using Skolem functions for representing dependent indefinite, as in (25), is less attractive since the descriptive content of the indefinite does not appear at the level of logical form, but only at the definition of the Skolem function. This would mean that we need a more complex translation algorithm, which would distribute the linguistic material between logical form and the definition of functions.

However, combining Skolem functions with indexed epsilon terms yields a much better representation. The indexed epsilon terms preserve the structure of NPs at the
level of logical form while Skolem functions between the indices represent the dependency structure. This is illustrated by the logical representation (28a) for the reading of (24a) with a dependent indefinite NP. In (28a), the term *every lion* is represented by a universal quantifier over epsilon indices and an epsilon term according to the equivalence (27) (cf. von Heusinger 1997a, von Stechow (this volume)). The representation (28a), where the universal quantifier for indices has wide scope with respect to the existential one, is equivalent to (28b), where the second choice function is determined by the Skolem function *f* and the value of the first choice function *i*. Thus this represents the dependency of the choice of the referent for *a zebra* from the choice of a particular lion. The second choice clearly depends on the first one, which is formally represented by the Skolem function from choice functions into choice functions.11 The reading in which the choice of the referent for *a zebra* is not dependent on the particular choice of a referent for *every lion* is represented in (29):

(27) \( \forall i \, G_i x \, F x \equiv \forall x [Fx \rightarrow Gx] \) for \( F \neq \emptyset \)

(28) a. \( \forall i \, \exists j [\text{eat}(\epsilon_i x \, \text{lion}(x)), \epsilon_j y \, \text{zebra}(y)] \]
   b. \( \forall i [\text{eat}(\epsilon_i x \, \text{lion}(x)), \epsilon_f j y \, \text{zebra}(y)] \]

\( f \): Skolem function from choice functions into choice functions

(29) \( \exists j \, \forall i [\text{eat}(\epsilon_i x \, \text{lion}(x)), \epsilon_f j y \, \text{zebra}(y)] \]

The representation of (24a) by (28b) and (29) with indexed epsilon terms provides a uniform logical form of the two readings of the indefinite. The difference lies in the anchoring of the index: in the dependent reading the index is determined by a Skolem function, while in the independent reading it is existentially quantified at the sentence level.

4 Choice functions and dynamic semantics

In the dynamic semantics with choice functions of Peregrin & von Heusinger (1995), or the Salience Change Semantics of von Heusinger (1997a), indefinite NPs introduce updates of choice functions. Definite and indefinite NPs are interpreted according to an input choice function \( \Phi \) that can be understood as standing for the accessibility structure of a discourse. Definite NPs receive their referents by applying this choice function to the set that is described by their descriptive content, i.e. the input choice function corresponds to the contextual given choice function \( \Phi \) mentioned in (21) above. Indefinite NPs, however, are assigned their referents by a newly introduced choice function \( \Phi_i \). But once an indefinite has been assigned its referent, the input choice function \( \Phi \) is updated to \( \Phi' \) with respect to the assignment of a value to the set described by the indefinite. The updated choice function \( \Phi' \) assigns the referent of the indefinite to the set described by the indefinite.

This is illustrated by (30a), the logical form (30b) and its interpretation (30c). The truth conditions (i) require that there are two choice functions \( \Phi_1 \) and \( \Phi_2 \) such that they assign a painter and a village to the set of painters and the set of villages, respectively. Furthermore, the pair of these two individuals are in the extension of the predicate *live*. The context change potential (ii) of the sentence updates the input choice function
\( \Phi \) twice: the first indefinite introduces the update function \( u_1 \) that changes the input choice function to \( \Phi' \) that differ from \( \Phi \) at most in the assignment to the set of farmers, which is the painter picked out by the choice function \( \Phi_1 \). The second indefinite introduces an update \( u_2 \) that modifies the given choice function \( \Phi' \) for the assignment to the set of villages. We can (informally) simplify the interpretation (30c) to (30d). Since the updated choice function \( u_1(\Phi) = \Phi' \) assigns the same painter to the set of painters as the choice function \( \Phi_1 \), we replace it by \( \Phi' \). Hence, the update function \( u_1 \) has two functions: assigning a value to the indefinite (by using a new choice function) and by updating the input choice function. In the remainder we will keep to this simplified interpretation mechanism.

(30)  
   a. \( \exists \) live(\( \epsilon_x \) painter(\( x \)), \( \epsilon_y \) village(\( y \)))  
   b. \( \exists \) live(\( \epsilon_x \) painter(\( x \)), \( \epsilon_y \) village(\( y \)))  
   c. \( \exists \) live(\( \epsilon_x \) painter(\( x \)), \( \epsilon_y \) village(\( y \)))  
      = 1 iff  
      (i) there are two choice function \( \Phi_1 \) and \( \Phi_2 \) such that  
          \( \langle \Phi_1(\[ \text{painter} \]), \Phi_2(\[ \text{village} \]) \rangle \in \[ \text{live} \] \) and  
      (ii) there are two update functions \( u_1, u_2 \) with \( u_1(\Phi) = \Phi' \) and \( u_2(\Phi') = \Phi'' \) such that \( \langle \Phi'(\[ \text{painter} \]), \Phi''(\[ \text{village} \]) \rangle \in \[ \text{live} \] \)  
   d. \( \exists \) live(\( \epsilon_x \) painter(\( x \)), \( \epsilon_y \) village(\( y \)))  
      = 1 iff there are two update functions \( u_1, u_2 \) with \( u_1(\Phi) = \Phi' \) and \( u_2(\Phi') = \Phi'' \) such that  
          \( \langle \Phi'(\[ \text{painter} \]), \Phi''(\[ \text{village} \]) \rangle \in \[ \text{live} \] \)  

The updated choice function \( \Phi'' \) is passed to the next sentence, and definite expressions can be interpreted according to this modified choice function. This is illustrated by the example (31), where definite expression are represented as epsilon terms with \( c \) as their index (cf. (21)). The definite NP the farmer denotes the object that is assigned by the input choice function \( \Phi \), whereas the indefinite NP a donkey refers to a donkey that was assigned by a modified choice function \( \Phi_1 \), or rather by the updated choice function \( \Phi' \) that differs from \( \Phi \) at most in the assignment of a certain donkey to the set of donkeys. Therefore, the subsequent definite NP the donkey can refer to this donkey by applying \( \Phi' \) to the set of donkeys. The updated choice function \( \Phi' \) assigns to the set of farmers the same farmer that was assigned by the original choice function \( \Phi \) since the update \( u_1 \) has only changed the value for the set of donkeys.

(31)  
   a. The farmer owns a donkey. The farmer feeds the donkey.  
   b. \( \exists \) own(\( \epsilon_x \) farmer(\( x \)), \( \epsilon_y \) donkey(\( y \)))  
      &  
      feed(\( \epsilon_x \) farmer(\( x \)), \( \epsilon_y \) donkey(\( y \)))  
   c. \( \exists \) own(\( \epsilon_x \) farmer(\( x \)), \( \epsilon_y \) donkey(\( y \)))  
      &  
      feed(\( \epsilon_x \) farmer(\( x \)), \( \epsilon_y \) donkey(\( y \)))  
      = 1 iff there is a choice function update \( u_1 \) with \( u_1(\Phi) = \Phi' \) and \( \langle \Phi'(\[ \text{farmer} \]), \Phi'(\[ \text{donkey} \]) \rangle \in \[ \text{own} \] \) and  
      \( \langle \Phi'(\[ \text{farmer} \]), \Phi'(\[ \text{donkey} \]) \rangle \in \[ \text{feed} \] \)
In the remainder, I mark the updated choice functions with the modification caused by the update function, i.e., by a new value $d$ with respect to a set $s$: $\Phi^{[s,d]}$. For example, the indefinite NP *a donkey* updates a given choice function $\Phi$ to $\Phi^{[\text{donkey},e]}$, where $e$ is the value assigned to the set of donkeys by this updated choice function.

### 4.1 Choice function updates and scope relations

Universal quantifiers are interpreted as introducing an update choice function which is restricted to the scope of the quantifier. This reflects their static behavior with respect to anaphoric expression outside of the scope, as illustrated in (32a). Thus, the two readings of the first sentence of (32a) are represented as (32b) and (32c) (cf. (28b) and (29), respectively):

(32) a. Every lion ate a zebra. *He liked it.
   b. $\forall i \left[ \text{eat}(\epsilon_x, \text{lion}(x), \epsilon_{f(i)} \text{zebra}(y)) \right]$
   c. $\exists j \forall i \left[ \text{eat}(\epsilon_x, \text{lion}(x), \epsilon_{j,y} \text{zebra}(y)) \right]$

The interpretation of the dependent (or narrow scope) reading of the indefinite (32b) is given in (32d). The Skolem function $f$ from choice function into choice functions represents the dependency of the choice of the referent for the indefinite from the choice for the universal expression. For all updated choice functions with respect to the value for lions there is a modified choice function with respect to the value for the set of zebras. The independent (or wide scope reading) of the indefinite (32c) is interpreted in (32e). Here the input choice function $\Phi$ is first updated by the indefinite NP and then the updated by the universal phrase *every lion*. Thus, the choice of the referent for *a zebra* does not depend on the choice of a particular lion, it is rather the same zebra for all lions.

(32) d. $\left[ \forall i \left[ \text{eat}(\epsilon_x, \text{lion}(x), \epsilon_{f(i)} \text{zebra}(y)) \right] \right] = 1$ iff there is a Skolem function $f$ from choice functions into choice functions such that for all update functions $u_1$ with $u_1(\Phi) = \Phi' = \Phi^{[\text{lion},b]}$ and $f(\Phi') = \Phi'' = \Phi^{[\text{lion},b]}[\text{zebra},r]$ such that $\langle \Phi'(\text{lion}), \Phi''(\text{zebra}) \rangle \in \text{eat} \rangle$

e. $\left[ \exists j \forall i \left[ \text{eat}(\epsilon_x, \text{lion}(x), \epsilon_{j,y} \text{zebra}(y)) \right] \right] = 1$ iff there is an update function $u_1$ with $u_1(\Phi) = \Phi' = \Phi^{[\text{zebra},r]}$ such that for all update functions $u_2$ with $u_2(\Phi') = \Phi'' = \Phi^{[\text{zebra},r]}[\text{lion},b]$ such that $\langle \Phi'(\text{lion}), \Phi''(\text{zebra}) \rangle \in \text{eat} \rangle$

Intermediate readings of indefinites in sentences with two operators, like (33a), can be analyzed in the following way. The update function introduced by the indefinite takes as its argument the choice function modified by the first operator, namely $\Phi^{[\text{prod},d]}$, yielding the new choice function $\Phi^{[\text{prod},d]}[\text{book},e]$. Thus the update caused by the indefinite *a book* does not depend on the update by the universal *every student*. This semantics reflects the intermediate scope reading of (33a).
(33) a. Every professor rewarded every student who read a book.
   b. For all updates $\Phi_1 = \Phi[([\text{prof}],d)]$ there is an update $\Phi_2 = \Phi[([\text{prof}],d)][([\text{book},e])]$ such that for all updates $\Phi_3 = \Phi[([\text{prof}],d)][([\text{book},e])][([\text{student}],h)]$.
   c. $\langle \Phi_1([\text{prof}]), \Phi_2([\text{student}]), \Phi_3([\text{book}]) \rangle \in \text{read}$

4.2 Dependency and symmetry

The formalism allows us to encode dependencies not only between indefinites and other operators but also between two or more indefinite NPs. For example, (34a) can be assigned three readings: one in which both indefinites are independent and two other readings in which one indefinite depends on the other indefinite. Dependent indefinites do not introduce updates on choice functions but they are characterized by Skolem functions from choice functions into choice functions. In the interpretation (34c), both indefinites introduce an update function from choice functions into choice functions. The interpretation (34e) of the logical form (34d) assume one update function introduced by the indefinite a painter. The second indefinite a village is interpreted by the Skolem function $f$ from the updated choice function $\Phi_1$ to the choice function $\Phi_1$ expressing the dependent nature of the second indefinite. (34g) expresses the opposite dependency structure:

(34) a. A painter lives in a village.
   b. live$(\varepsilon, x, \text{painter}(x), \varepsilon, y, \text{village}(y))$
   c. $||\text{live}(\varepsilon, x, \text{painter}(x), \varepsilon, y, \text{village}(y))|| = 1$ iff there are two choice function updates $u_1$ and $u_2$ with $u_1(\Phi) = \Phi[([\text{painter}, d])] = \Phi_1$ and $u_2(\Phi) = \Phi[([\text{village}, f])] = \Phi_2$ such that $\langle \Phi_1([\text{painter}]), \Phi_2([\text{village}]) \rangle \in \text{live}$
   d. live$(\varepsilon, x, \text{painter}(x), \varepsilon, f(i,y), \text{village}(y))$
   e. $||\text{live}(\varepsilon, x, \text{painter}(x), \varepsilon, f(i,y), \text{village}(y))|| = 1$ iff there is a Skolem function $f$ and an update $u_1$ with $u_1(\Phi) = \Phi[([\text{painter}, d])] = \Phi_1$ and $f(\Phi_1) = \Phi[([\text{village}, f])] = \Phi_2$ such that $\langle \Phi_1([\text{painter}]), \Phi_2([\text{village}]) \rangle \in \text{live}$
   f. live$(\varepsilon, f(i), x, \text{painter}(x), \varepsilon, f(i), y, \text{village}(y))$
   g. $||\text{live}(\varepsilon, f(i), x, \text{painter}(x), \varepsilon, f(i), y, \text{village}(y))|| = 1$ iff there is a Skolem function $f$ and an update $u_1$ with $u_1(\Phi) = \Phi[([\text{village}, f])] = \Phi_1$ and $f(\Phi_1) = \Phi[([\text{painter}, d])] = \Phi_2$ such that $\langle \Phi_2([\text{painter}]), \Phi_1([\text{village}]) \rangle \in \text{live}$

Although these representations do not show any truth conditional effects, combined with other operators, like conditionals and adverbs of quantification, they yield different truth conditions accounting for the difference between symmetric and asymmetric readings discussed in section 2.4. Sentence (35a) has a prominent symmetric reading, and therefore receives the representation (35b) where the unselectively binding operator $\forall$ binds all discourse referents. This is equivalent to the classical formula (35c). The epsilon representation (35d) with the universal quantifier over indices, i.e. choice
function variables, is equivalent to (35b) and (35c) if there are hunters and zebras. All three formulae are instances of the so called ‘strong’ reading of a donkey sentence since they universally quantify over both variables.

(35) a. If a hunter sees a zebra, he chases it.
   b. \( \forall (x, y \mid \text{hunter}(x) \& \text{zebra}(y) \& \text{see}(x, y), [x, y \mid \text{chase}(x, y)]) \)
   c. \( \forall x \forall y \left[ (\text{hunter}(x) \& \text{zebra}(y) \& \text{see}(x, y) \rightarrow \text{chase}(x, y)) \right] \)
   d. \( \forall \left[ \text{see} \varepsilon_i x \text{hunter}(x), \varepsilon_k y \text{zebra}(y) \rightarrow \text{chase}(\varepsilon_i x \text{hunter}(x), \varepsilon_k y \text{zebra}(y)) \right] \)

(16a), repeated as (36a), exhibits an asymmetric reading, i.e. we must count donkey-owning farmers rather than farmer-donkey pairs. In the epsilon analysis, we assume that we first make a choice of a farmer on which the choice of the donkey depends: The first choice constrains the set of possible candidates for the second choice. The Skolem function ties the choice of a donkey to the choice of the farmer and it expresses a quasi uniqueness condition on the donkey. The result of this is that we must only consider choice functions that vary in the assignments for farmers. If we apply an unselectively binding operator as MOST in (36b), it can only bind the one free index yielding the intuitively correct asymmetric reading, as expressed in the interpretation (36c):

(36) a. If a farmer owns a donkey he usually beats it.
   b. MOST\( \left\{ \text{own} \varepsilon_i x \text{farmer}(x), \varepsilon_f y \text{donkey}(y) \right\}, \text{beat} \varepsilon_i x \text{farmer}(x), \varepsilon_f y \text{donkey}(y) \right\} \)
   c. (36a) is true iff there are more choice function updates \( \Phi' = \Phi[\{\text{farmer}\}] \) that extend to \( \Phi'' = \Phi[\{\text{farmer}\}][\{\text{donkey}\}] \) such that
      \[ \langle \Phi'(\{\text{farmer}\}), \Phi''(\{\text{donkey}\}) \rangle \in \{\text{own}\} \quad \text{and} \]
      \[ \langle \Phi'(\{\text{farmer}\}), \Phi''(\{\text{donkey}\}) \rangle \in \{\text{beat}\}, \quad \text{than there are choice} \]
      \[ \text{function updates} \quad \Phi' = \Phi[\{\text{farmer}\}] \quad \text{that extend to} \quad \Phi'' = \]
      \[ \Phi[\{\text{farmer}\}][\{\text{donkey}\}] \quad \text{such that} \quad \langle \Phi'(\{\text{farmer}\}), \Phi''(\{\text{donkey}\}) \rangle \in \{\text{own}\} \quad \text{but} \]
      \[ \langle \Phi'(\{\text{farmer}\}), \Phi''(\{\text{donkey}\}) \rangle \not\in \{\text{beat}\} \]

5 CONCLUSION

I have discussed the contrast between different readings of indefinites, which triggered such theories as the ambiguity theory of Fodor & Sag (1982), and their reinterpretation by Kratzer (1998). I have argued that we can account for a uniform representation of indefinite NPs by means of indexed epsilon terms, which are interpreted as choice functions. The different readings of indefinite NPs are represented by different binding relations of the indices of the epsilon operator (or: by different binding relations of choice functions variables): dependent readings are represented by Skolem functions while independent readings are bound by some text operator. The representation of indefinite NPs is also extended to that of definite NPs, which are epsilon terms that depend on a contextually given choice function. Thus, this theory provides a uniform picture of indefinite and definite NPs. Indefinite NPs additionally introduce an update
function that modifies an input choice function with respect to the description of the indefinite. In this way anaphoric relations between indefinite NPs and definite NPs can be reconstructed without assuming co-indexing as in other theories. Finally, it was shown that the choice of a referent for an indefinite can also depend on the choice of a referent for another indefinite, yielding a dependency structure with one free choice function variable only. If such an structure is governed by some other operator, like an adverb of quantification, the corresponding operator can bind only one free choice function variable, yielding an asymmetric reading.

The theory presented here inherits the individualizing function of indefinites from the traditional grammarian view, but at the same time it can account for the variability that was one of the main motivation for the classical approach with quantifiers. Unlike the classical view and Lewis-Heim-Kamp theories, indefinites are represented as terms, rather than as quantifiers or as open sentences with associated variables (or discourse referents). In this way the descriptive material that identifies the referent can be distinguished from the descriptive material in the main predication or assertion of the sentence. The theory assumes with Farkas’ indexical theory of scope that the variability of indefinites is to be located at the interpretative level and not at the representative level. It disagrees with Farkas in the representation of the context change potential of indefinites. Farkas models it in terms of extensions of assignment functions, whereas the theory presented here assumes updates of choice functions. This is also the essential distinction from other choice function approaches (Reinhart, Kratzer, Winter, von Stechow), which propose that indefinites introduce arbitrary choice function, but which do not extend the semantics to a dynamic semantics that can capture anaphora, as well.12

NOTES
1 Behagel (1923, I, 45): “Der unbestimmte Artikel hat die Aufgabe, aus verschiedenen Vertretern einer Gattung einen einzelnen herauszuheben.” Engl. transl. provided by me.
2 Hemingway, Old Man at the Bridge, 1.
3 For earlier criticism of a substituting view with respect to anaphoric pronouns, see Egli (this volume) on the stoic conception of anaphora, and Hülsen (this volume) on the medieval discussion of anaphora.
6 Hemingway: Old Man at the Bridge, 1.
7 Hemingway: The Old Man and the Sea, 5.
8 Dynamic logic belongs to this tradition, too. See Groenendijk & Stokhof (this volume) for an overview.
Alternatively, we can assume that a choice function assigns a designated object “*” to the empty set such that any predication of it fails (cf. von Stechow (this volume)).

Chierchia (1992, 160) discusses the use of Skolem functions for E-type pronouns in general on the example (i) and its representation (ii):

(i) Every man who has a donkey beats it.
(ii) $\forall x [\text{man}(x) \land \exists y [\text{donkey}(y) \land \text{has}(x, y)]] \rightarrow \text{beat}(x, f(x)]$

Eventually he concludes that the function $f$ is a choice function: “It will have to be a function that maps each man into one of the donkeys he owns. It will be, thus, a choice function. And, consequently, it won’t in general be unique. This type of contexts will make salient not just one function but a family of functions, all of which are a priori good candidates (… ).”

This is the original treatment proposed in von Heusinger (1997a). Kratzer (1998, 168) gives a similar analysis according to which the Skolem function assigns each individual a (possibly) different choice function. She comments on her analysis (ii) of Hintikka’s (1986) example (i) as follows: “The complex determiner a certain is represented here as the free function variable $f$. Its implicit argument appears as a subscribed variable. Possible values for the variable $f$ are (partial) functions that map individuals into choice functions. In this particular example, the contextually determined value for the variable $f$ is a function that maps every husband into a choice function that is defined for just one argument, the set of all dates, and picks that man’s wife’s birthday from that set.”

(i) Each husband had forgotten a certain date – his wife’s birthday.
(ii) $\forall x [\text{husband}(x) \rightarrow \text{had forgotten}(x, f(x), date)]$

This paper constitutes the final part of a larger project analyzing the semantics of NPs and pronouns using indexed epsilon terms and choice functions (Egli 1991), which was financed by the Deutsche Forschungsgemeinschaft. In preceding papers it was first argued that Evans’ concept of E-type pronouns can be reconstructed by indexed epsilon terms (cf. Egli & von Heusinger 1995). Furthermore, it was shown that definite NPs in general must be represented as terms, in particular as context dependent epsilon terms (cf. von Heusinger 1997b). Their interpretation depends essentially on the accessibility structure of a given text or discourse. The final step is presented in this paper, showing that indefinites are represented by indexed epsilon terms and by update functions.

**References**


