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The Epsilon Operator and E-Type Pronouns

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0. *Introduction*

In this paper we attempt to provide a homogeneous description of definite and indefinite noun phrases. We therefore assume *one* semantic representation for *one* interpretation that the definite article *the* and the indefinite article *a* share. Our starting point is the classical theory of descriptions as developed by Russell in his famous article of 1905, "On Denoting."

In the first section we introduce the basic assumptions or intuitions that form the foundation for our considerations. Russell's analysis will be shown to be too restrictive to deal with natural language phenomena. The exposition of the intuitions will be more comprehensive than is usually the case, as we think that for a formalism devised for natural language description our basic ideas about language are at least as important as correctness and elegance in the formal model. Furthermore, it will become apparent that some other approaches to descriptions actually fail because they do not pay sufficient attention to basic characteristics of natural language.

In the second section a choice function, namely, the epsilon operator suggested by Hilbert, will be introduced and modified to capture the content of definite and indefinite descriptions adequately. Its properties will be compared with the basic ideas set forth in section 1. This will include a closer look at the well-ordering of sets and the dependency of descriptions on situations. In addition, we will introduce simple rules of thematization and rhematization that allow to specify the description or to extract a predicate from a description. These rules are largely guided by the actual use of natural language. It will become clear that the modified epsilon operator is much more appropriate to the variations of actual language use than Russell's approach with the iota operator.

In the third section we apply the analysis proposed in the second section by means of epsilon to E-type pronouns. We will show that our analysis solves the problematic bishop-sentences, since it does not imply any uniqueness condition. The ideas developed in this paper go back to Egli (1991) (in circulation since 1989).

1. *Basic Ideas in the Description of Definite and Indefinite NPs*

1.1 *Syntax and Semantics*

Referring expressions can be divided into proper names, such as *Hans, Constance, Mainau,* etc.; deictic expressions like *this, those over there, this island,* etc.; and descriptions like *the island on Lake Constance, the center of mass of the Solar System,* etc. One of the characteristics of referring expressions is that they always refer to exactly one individual, as opposed to, for example, predicates or generic expressions, which specify a property that might be true for one or more individuals or for none.

Proper names are invariable with respect to situations, whereas deictic expressions depend crucially on the circumstantial context. In the course of this paper the character of descriptions will turn out to lie between these two extremes. In natural language, definite as well as indefinite descriptions occur in the form of noun phrases. Or in other words, most interpretations of definite or indefinite NPs can be represented as descriptions. (For the difference between definite and indefinite descriptions, see section 1.7.) Syntactically they can be analyzed in the following way:

(1) definite article + common noun phrase, e.g.,
 a. the + island
 b. the + island that is in the sun
 c. the + blue pullover

From now on we will use the word *description* to refer to the whole noun phrase (e.g., *the island that is in the sun*) and the term *descriptive content* for the common noun phrase without the definite article (in our example, *island that is in the sun*). Semantically, the common noun phrase corresponds to an

open sentence "... x ... ." The linguistic content of the article should be represented by a constant which selects exactly one individual, namely, one that fits the descriptive content in the open sentence. This constant is a variable binding operator, as in (2). Accordingly, examples (1a) to (1c) can be rendered as (3) to (5).

(2) constant for article + variable + open sentence with bound variable
(3) the x, such that x is an island
(4) the x, such that x is an island and x is in the sun
(5) the x, such that x is a pullover and x is blue

Concerning the interaction of syntax and semantics, we will make two demands on a description of definite and indefinite noun phrases. First, a semantic representation should follow the surface structure as closely as possible. Furthermore, the meaning of a description must be derivable from the meanings of its parts, that is, from the meanings of the article and from the open sentence. This is the claim of semantic compositionality.

1.2 *The Assumption of Uniqueness and Existence*

It is widely supposed that a description denotes exactly one individual. This individual must be characterized by the descriptive content in the common noun phrase. If there is just one individual for which the description is true, as in (6), then this individual is sufficiently specified by the description. However, how is it to be determined what a description denotes if there is more than one individual that is correctly characterized by the description, as in (8), or if there is no such individual, as in (7)?

(6) the center of mass of the Solar System
(7) the present king of France
(8) the island on Lake Constance

Since Russell was only interested in descriptions like (6), he claimed that the descriptive content in a definite description has to be true for at least one individual (existence presupposition) and one individual at most (uniqueness presupposition). Both of these requirements are represented by the iota operator, which is used as a semantic representation for the definite article. Examples (6) to (8) can thus be formalized as (6a) to (8a):

(6) a. ιx [center of mass of the Solar System(x)]
(7) a. ιx [present king of France(x)]
(8) a. ιx [island on Lake Constance(x)]

For an indefinite description, Russell demands the existence presupposition but rejects the uniqueness presupposition. Linguistically, this is represented by the indefinite article *a*, not by the definite article *the*. According to Russell's theory, sentences containing descriptions that do not satisfy the existence and uniqueness presuppositions have the truth-value false. However, as sentences with such descriptions occur frequently in natural language, a linguistic analysis must take them into consideration.

Let us first consider the uniqueness presupposition as it relates to examples (9) and (10):

(9) An island on Lake Constance is in the sun.
(10) The town on Lake Constance is famous.

In both (9) and (10), there is exactly one individual denoted by the description, although there are several islands on Lake Constance, just as there is more than one famous town on Lake Constance. This demonstrates two things: The uniqueness presupposition, that is, the claim that the descriptive content may only be true for one individual, is a useful property, because in a such restricted case the description is specified solely by the descriptive content. But it is not a necessary property of descriptions, since the individual that the description is meant to denote, as we will show, is specified not only by the descriptive content but also by the semantic representation for the article. From the supposed fact that a description denotes exactly one individual, Russell concluded that the uniqueness presupposition was a valid restriction of the descriptive content within the description. However, we assume instead that uniqueness is introduced via the context and only shown by use of the article and that the descriptive content can indeed be true for more than one individual. Russell's assumption, then, is a special case, describing exactly that situation where the descriptive content is true for only one individual.

The second point shown by the counterexamples (9) and (10) is that the difference between definite and indefinite descriptions does not, as Russell assumed, lie in the uniqueness presupposition, but in the contrast between already known versus newly introduced (cf. section 1.7). This is so because both descriptions denote exactly one individual, although the respective descriptive contents are correct for more than one individual.

The question of whether there is at least one object satisfying the descriptive content (existence presupposition) can be divided into two problems: the problem of nonexistent objects, as shown in (11), and the problem of impossible objects, as in (12):

(11) The flying horse is white.
(12) The round square is round.

Nonexistent objects can be readily integrated into a semantics. The problem of impossible objects, first raised by Meinong, is more difficult to judge. With regard to sentences like (12), our linguistic intuition seems to fail. Interestingly enough, we do find instances of such impossible descriptions in our speech. For example, the impossible description in (14) is hidden in sentence (13):

(13) A wet nappy is not wet because no nappy is wet.
(14) the nappy that is wet and not wet

At least one interpretation allows an impossible description like this one to denote generality (in this example, that all nappies are not wet). This interesting observation points to a linguistic borderline case that will be reflected in a corresponding formalization later on (cf. section 2.2).

1.3 *Order and the Hierarchy of Salience*

The descriptive content in a description might apply to several individuals (cf. example (9)); however, a description always denotes only one individual, namely, the most salient individual or the first one within a linear order. We thus have to assume that those individuals that share a correct descriptive content are well-ordered. Following Lewis (1979), we can think of this order as a hierarchy of salience that is valid in a particular situation (cf. section 1.4).

In natural language this sort of order can be represented by expressions like *the first mentioned, the second mentioned*; *the first, the second*; *the one, the other,* etc.

Let us start with the assumption that there are only three islands on Lake Constance: Mainau, Reichenau, and Lindau; and let us also assume that in some situation they are graded in this very order. Then we can use the above-named expressions to describe each element of the well-ordered set of islands. Here also contextual factors have a part to play. In our example we refer to the island Mainau by (15a), to the island Reichenau by (15b) and to the island Lindau by (15c).

(15) The islands on Lake Constance: Mainau, Reichenau, Lindau
a. *the* island, *the first* island, *the one* island, *this* island
b. *the second* island, *the other* island, *that* island, *the last* island *but one*
c. *the third* island, *the further* island, *that* island *over there*, *the last* island

1.4 *Context Dependency of Descriptions*

In the last section we argued that a description always denotes the most salient element of a set given by the descriptive content. Which of the individuals in a well-ordered set is actually the first depends on various factors, most especially on the current situation. Every object can be made the most salient one within a particular context. So, for example, the description *the island*, used in a situation like standing on the terrace of Constance University (looking down at the island of Mainau) will probably denote Mainau. We can say that an order in which Mainau is the first element has been established by the situation. A different situation, like strolling on the island Reichenau, will establish a different order where Reichenau is the first element and the description *the island on Lake Constance* denotes Reichenau.

Despite the effects of context dependency, an individual can be specified independently of particular situations. So, for instance, even while walking on Reichenau, we could refer to Mainau simply as *the island*. There would merely need to exist an established order of which the first element is the island Mainau.

1.5 *The Specialization of Descriptions*

In natural language we find many cases where more than one object fall under the descriptive content of a (improper) description as in (9) and (10). In order to save the uniqueness condition, this fact is often handled by reducing the domain of relevant objects to the unique object that is referred to by the description. This domain is called *the minimal situation* (Heim 1990; Kadmon 1990). This seems a bit artificial in two ways. First, the genuine property of a description referring to a unique object becomes trivial since the domain is such that there is only one object that can be referred to. Uniqueness is then a property of a certain situation and not of a description. Second, there are no rules for constructing such minimal situations.

Our approach is quite different. We assume that there can be many objects falling under the descriptive content of a given description. Which

object is referred to depends on the context. Still, it is possible to include some more properties of the context in the description. Specializing the description in this way reduces the number of objects that fall under the descriptive content, or in other words, it makes the description less context dependent. In the extreme case, the description is so specialized that only one object falls under the descriptive content. This is the Russellian case in which the uniqueness condition on the objects singled out by the descriptive content is fulfilled. We can account for one type of specialization, namely, the incorporation into a given description of an additional property that is predicated of this description. Thus, we derive the extended description (16a) from the sentence (16).

(16) The island on Lake Constance can be seen from the university.
a. the island on Lake Constance that can be seen from the university

The descriptive content of the subject noun phrase in (16) comprehends three individuals. However, when this descriptive content is specified as in (16a) by the relative clause *that can be seen from the university*, the possibilities for identifying the intended referent of the phrase are reduced to one, and this is the island Mainau. Such an approach seems more natural than that of reducing contexts to minimal situations. In section 2.5, we give some formal rules for deriving this specialization and its inversion.

1.6 *Maxim of Exactness*

The endeavor to use a description that refers to an individual unambiguously can be understood as a conversational maxim, that of exactness, after Grice: *Be as unambiguous as possible and not vacuous.* We can observe this maxim, as Russell suggests (cf. section 2.1), by giving a descriptive content that is true for exactly one individual. But we could also narrow the descriptive content in the course of a conversation until only one individual in the given situation is left. Finally, it is possible to pick out the most salient individual of the ordered set falling under the descriptive content, according to the semantics of the definite article developed in this paper. This strategy is the most general of the three mentioned. The other two can be understood as special cases. Thus we have different strategies for fulfilling the maxim of exactness, although these are not necessarily mutually exclusive and might be used in combination.

1.7 *Definiteness*

So far we have dealt with definite and indefinite noun phrases in the same way, as being definite and indefinite descriptions. This reflects the attempt to preserve in the semantic analysis the syntactic categorization they have in common. However, we have to pose the question of the distinguishing feature of definiteness. Russell saw the difference in the uniqueness presupposition that was necessary for definite descriptions but not for indefinite descriptions. But since we have shown in section 1.2 that Russell's uniqueness presupposition does not account for the actual distinction in natural language, this feature can be abandoned.

Instead, we want to assume following Heim (1982) that the crucial difference between indefinite and definite NPs is the one between new and already known (cf. (9) and (10)). A definite NP is needed if the denoted individual or the discourse referent has already been introduced. As shown in (17), the same descriptive content can then be taken up again.

(17) A man enters. The man sits down.

Of course, the discourse referent can also be introduced by a description with a different descriptive content from the one we use later to refer back to it, as in (18):

(18) The president enters. The first executive sits down.

Unlike both these uses of descriptions, which can be called anaphoric, there is another way of using definite descriptions, which is usually to be understood as deictic; see (19):

(19) The island is nice.

We can utter (19) without having introduced the discourse referent explicitly. It has to be inferable, however, from the situation or context.

An indefinite NP is always needed if a new discourse referent is introduced for the first time, as in the first sentence of (17). An indefinite NP therefore can never refer to a discourse referent already introduced. Thus the two descriptions *a man* in (20a) denote two different men. Consequently, (20a) is equivalent to (20b), unlike (21a) and (21b), since the sentences in (21a) refer to the same man whereas those in (21b) refer to two different men.

(20) a. A man enters the room. A man leaves the room.
b. A man enters the room. An*other* man leaves the room.

(21) a. The man enters the room. The man leaves the room.
b. The man enters the room. The *other* man leaves the room.

2. *The Epsilon Operator*

In the last section we presented our intuitions concerning definite and indefinite NPs and made clear as well that Russell's analysis does not suffice to account for natural language descriptions. Egli (1991) has suggested using the epsilon operator for the representation of descriptions instead of the iota operator. The epsilon operator can be viewed as a generalized iota operator without existence presupposition and uniqueness presupposition.[1] Hilbert introduced the epsilon operator as an indeterminate symbol that assigns an element to a given set, but does not specify which element. The advantage of assuming an indeterminate symbol is that the universal quantifier and the existential quantifier can be defined by the epsilon operator and we can therefore derive for each formula a quantifier-free equivalent. Also, the epsilon operator is closely connected to the axiom of choice (cf. Leisenring 1969:105ff.). The disputed status of the axiom of choice, as well as the indeterminism, has caused a certain lack of appreciation of the epsilon operator. We will try to provide a well-motivated description to make the benefits of the epsilon operator entirely clear.

2.1 *Syntax and Semantics of Hilbert's Epsilon Operator*

The epsilon operator corresponds to a choice function that assigns to each nonempty set one element of this set. An empty set will be assigned an arbitrary element. Thus it is guaranteed that an expression like εx Fx under all circumstances denotes something and that there are no cases where it has no reference as in Russell's system.[2] Like the iota operator, the epsilon operator forms a term (constant) from a sentential form. Unlike the iota operator, it carries no existence or uniqueness presupposition with it. The main difference may be shown by the formalization and the paraphrase of the description *the island*, as given in (22) and (23):

(22) ιx [island(x)]
the **unique** x, such that x is an island

(23) εx [island(x)]
the **selected** x, such that x is an island

To introduce epsilon terms into a first-order predicate logic, we will adopt the following axiom by Hilbert and Bernays, which they call the epsilon formula:

(24) epsilon formula: $Fa \rightarrow F\,\varepsilon x\, Fx$

From each formula of the form Fa we can go directly to the corresponding formula $F\varepsilon x\, Fx$. The only new constant that has to be introduced is the symbol ε. This can be done equivalently by means of the following rules, which we call the first and the second Hilbert rule (Hilbert & Bernays 1939:15):

(25) first Hilbert rule: $\exists x\, Fx \equiv F\,\varepsilon x\, Fx$
(26) second Hilbert rule: $\forall x\, Fx \equiv F\,\varepsilon x\, \neg Fx$

The content of these two equivalences can be conceived as follows. If there is an x that has the property F, then there is also a description $\varepsilon x\, Fx$ that satisfies F. To look at it the other way around: if there is a description $\varepsilon x\, Fx$ that satisfies F, then there must be as well an x that satisfies F. The intuitions behind the second Hilbert rule are a bit more difficult: $\neg F$ denotes the complement of the set F. By $\varepsilon x\, \neg Fx$ this complement is assigned one element, which either is an element of $\neg F$, if $\neg F$ is not empty, or is an arbitrary (but constant) element, if $\neg F$ is empty. Precisely the latter, however, has to be the case, as $\varepsilon x\, \neg Fx$ is an element of F. Therefore, $\neg F$ must be empty, and thus F denotes generality or: $\forall x\, Fx$. The second Hilbert rule can be derived from the first by replacing F by $\neg F$ and invoking contraposition.

The epsilon formula and the two Hilbert formulae were given by Hilbert and Bernays only for monadic predicates as presented above. However, we will also have to deal with complex formulae. In that case, the general formulation for an arbitrary formula α will be as follows:

(24*) epsilon formula: $\alpha \rightarrow \alpha\,(^{x}/\varepsilon x\, \alpha)$
(25*) first Hilbert rule: $\exists x\, \alpha \equiv \alpha\,(^{x}/\varepsilon x\, \alpha)$
(26*) second Hilbert rule: $\forall x\, \alpha \equiv \alpha\,(^{x}/\varepsilon x\, \neg\alpha)$

Hilbert and Bernays did not give a semantic interpretation of their epsilon symbol,[3] leaving this task for others. Schröter (1956:59) proposed interpreting the epsilon operator by a choice function. Asser (1957) then formulated this idea in the necessary detail. Following Asser we will interpret the epsilon operator by a choice function Φ, which assigns one of its elements to each non-empty set and an arbitrary element to the empty set.[4]

We have to extend a model $\mathfrak{M}$ by the choice function Φ. Then we get the triple $\langle A, I, \Phi\rangle = \mathfrak{M}$ with the domain of discourse A, an interpretation I of the constants, and the choice function Φ. Also we have an assignment g of individuals to the variables as usual. The interpretation of an epsilon term $\varepsilon x\, \alpha$ is given by the following rule: $[[\varepsilon x\, \alpha]]^{\mathfrak{M},g} = \Phi(M_i)$, where M_i is the set of individuals $\{a\colon [[\,\alpha\,]]^{\mathfrak{M},gx/a} = 1\}$.

(27) $[[\varepsilon x\, \alpha]]^{\mathfrak{M},g} = \Phi(\{a\colon [[\,\alpha\,]]^{\mathfrak{M},gx/a} = 1\})$

Let us return to our example of the Lake Constance islands to have a closer look at this. Let Φ_1 be the choice function that assigns an individual, namely Mainau, to the set of islands, thus:

(28) $\Phi_1(\{\text{Mainau, Reichenau, Lindau}\}) = \text{Mainau}$

Now we can give the interpretation of the term *εx island(x)* that stands for *the island*:

(29) $[[\varepsilon x\ \text{island}(x)]]^{\mathfrak{M},g} = \Phi_1(\{a\colon [[\text{island}(x)]]^{\mathfrak{M},gx/a} = 1\})$
$= \Phi_1(\{\text{Mainau, Reichenau, Lindau}\})$
$= \text{Mainau}$

The sentence *The island is nice*, which contains a description, can now be interpreted by the given choice function Φ_1 as follows:

(30) $[[\text{nice}\ (\varepsilon x\ \text{island}(x))]]^{\mathfrak{M},g} = 1$
iff $[[\text{nice}]]^{\mathfrak{M},g}\,([[\varepsilon x\ \text{island}(x)]]^{\mathfrak{M},g}) = 1$
iff $[[\text{nice}]]^{\mathfrak{M},g}\,(\Phi_1(\{a\colon [[\text{island}\ (x)]]^{\mathfrak{M},gx/a} = 1\})) = 1$
iff $[[\text{nice}]]^{\mathfrak{M},g}\,(\text{Mainau}) = 1$
iff Mainau belongs to the set of nice things.

In a complex formula Ψ with a description $\varepsilon x\, \alpha$ the chosen element a has to make the formula true. For a nonempty property, then, the chosen individual has to fulfill the property given in the description, as well as the property given in the formula. If the description $\varepsilon x\, \alpha$ from (31) has the value a, i.e. $[[\varepsilon x\, \alpha]]^{\mathfrak{M},g} = a$, then (31a) and (31b) are valid.

(31) $[[\Psi(x/\varepsilon x\, \alpha)]]^{\mathfrak{M},g} = 1$ iff $[[\Psi]]^{\mathfrak{M},gx/[[\varepsilon x\, \alpha]]^{\mathfrak{M},g}} = 1$
a. iff $[[\alpha]]^{\mathfrak{M},gx/a} = 1$ and $[[\Psi]]^{\mathfrak{M},gx/a} = 1$
b. iff $[[\alpha\ \&\ \Psi]]^{\mathfrak{M},gx/a} = 1$

2.2 *Uniqueness and Existence Presuppositions for the Epsilon Operator*

The epsilon operator does not carry with it any presupposition of uniqueness. For this reason, it is more flexible and more suitable for the description of natural language than Russell's representation, which stipulates the uniqueness of the individual to which the common noun phrase applies. The question of whether one has to presuppose the existence of at least one object with the property given in the description is more difficult to answer. An epsilon term is defined as assigning an arbitrary individual to the empty set. Slater (1988) uses epsilon expressions to describe sentences with nonexistent objects. For example, (32) can be uttered reasonably without producing a contradiction. The description *the ghost in the attic* cannot be expressed within the classical theory of description, as it violates the presupposition of existence. However, under our present view it can be uttered, as shown in (32a), to express the nonexistence of an object.

(32) The ghost that is noisy in the attic is not a ghost.
a. $\neg G\, \varepsilon x\, [Gx\, \&\, Nx]$

Under the definition of the epsilon operator, the description *the ghost that is noisy in the attic* or $\varepsilon x\, [Gx\, \&\, Nx]$ denotes an arbitrary object because the set $(Gx\, \&\, Nx)$ is empty. So, for example, it could be a cat or an open window that rumbles in the attic. From (32a) the truth of (32c) follows by the tautology $\neg p \rightarrow \neg(p\, \&\, q)$ via the intermediate step (32b) and by the first Hilbert rule.

(32) b. $\neg(G\, \varepsilon x\, [Gx\, \&\, Nx]\, \&\, N\, \varepsilon x\, [Gx\, \&\, Nx])$
c. $\neg(\exists x\, [Gx\, \&\, Nx])$

Impossible objects, like the ones we mentioned in section 1.2, can be represented as epsilon terms as well. Thus we can transform (14) into sentence (33) and formalize it as in (33a). By means of propositional calculus we derive (33b), this time using the propositional simplification rule $p \rightarrow (q \rightarrow p)$. Example (33b) may be transformed into (33c) and, by the second Hilbert rule, is equivalent to (33d).

(33) The nappy that is not dry is dry.
a. $D\, \varepsilon x\, [Nx\, \&\, \neg Dx]$
b. $N\, \varepsilon x\, [Nx\, \&\, \neg Dx] \rightarrow D\, \varepsilon x\, [Nx\, \&\, \neg Dx]$
c. $N\, \varepsilon x\, \neg[Nx \rightarrow Dx] \rightarrow D\, \varepsilon x\, \neg[Nx \rightarrow Dx]$
d. $\forall x\, [Nx \rightarrow Dx]$

The set or property $(Nx\, \&\, \neg Dx)$ or $\neg(Nx \rightarrow Dx)$ must be empty for the formula (33a) to be true. In that case, however, their negation, that is, $(Nx \rightarrow Dx)$, denotes the universal set. And, although the sentence seemed a bit odd at first sight, this is exactly the generalization intended.

A similar case can be shown by a classical expression from Greek literature used by the poet Kallimachos (4./3. cent. B.C.) (cf. Egli 1991:18). Diodoros was a logician who was — at least ironically speaking — beyond reproach. Kallimachos therefore let the god of reprehension write on the wall that Diodoros was wise. He insinuated thus that (even) the one who reprehends all does not reprehend Diodoros. This statement means much the same thing as the sentence in (34), which can be represented formally by (34a):

(34) Even the one who reprehends Diodoros does not reprehend him.
a. $\neg R(\varepsilon x\, [R(x, d)], d)$
b. $\neg\exists x\, R(x, d)$

According to the first Hilbert rule (25), (34a) is equivalent to (34b), which means of course that nobody reprehends Diodoros. These examples have shown that the definition of the epsilon operator for empty sets allows us to analyze sentences that could not be represented within more classical formats. Moreover, the formalization using epsilon terms is always very close to the surface of natural language expressions.

2.3 *Well-ordering and Epsilon Terms*

Every choice function imposes an order on every nameable set, which can be viewed as a hierarchy of salience in the sense of Lewis (1979). The description $\varepsilon x\, Fx$ denotes the most salient individual that has the property F. In the following we shall not be concerned with empty sets. The intuitive order of the individuals that fall under a certain description can be reconstructed logically by a recursive definition and the use of the epsilon operator within the semantic reconstruction of natural language phenomena. This recursive definition not only indicates that every set can be well-ordered, but also specifies the meaning of the intuitive natural language ordinal numbers *the first, the second, the third,* etc. and similar expressions like *the other, the further,* etc. These reconstructions of natural language ordinal numbers should be kept separate from von Neumann's set theoretical reconstructions of ordinal numbers, very much as the von Neumann cardinal numbers should be kept distinct from the generalized quantifiers representing the numbers in the tradition of Frege.

Thus, for example, the expression *the island* or *the first island* denotes the most salient, the most prominent, the most conspicuous island that can be talked about in a given situation. We describe this by εx island(x). This expression denotes the island chosen first. *The second island* or *the other island* denotes the most salient of all the islands except the first: εy [island(y) & $y \neq \varepsilon x$ island (x)]. The expression within brackets corresponds to the set theoretical subtraction: $\{a: Fa\} \setminus \{\Phi(\{a: Fa\})\}$. This can be continued. By simultaneous induction we can define the expressions *the n-th F* and *the first n F*. The expression *the n+1-th island* denotes the island that is on the n+1-th position in the hierarchy of salience, or the chosen island that is not identical with any of the n islands chosen first. This is done in definition (35). In terms of natural language, the expression in clause (i) sounds a bit artificial (following Egli 1991:17).

(35) (i) the first 1 F = λx ($x = \varepsilon x$ Fx)
(ii) the first n+1 F = λx ($x = \varepsilon x$ [Fx and x is not one of the first n F] or x is one of the first n F)
(iii) the 1st F = εx Fx
(iv) the n+1-th F = εx [Fx and x is not one of the first n F]

The meaning of the ordinal numbers can be obtained from (35) by abstracting away the property F:

(35) a. (i) the first = λF the 1st F = λF (εx Fx)
(ii) the nth = λF the n-th F

Expressions like *the*, *the first*, *this* yield the meaning (35a(i)), whereas expressions like *the other*, *the second*, *that* after (ii) get the following meaning: $\lambda F(\varepsilon x$ [Fx and x is not the first F]).

2.4 *Dependency of the Epsilon Operator on Situations*

Descriptions are situation dependent, as we showed in 1.4. We build this dependency into our notation by assuming not *one single* choice function but *a whole family* of them indexed with situations. According to our exposition in the last section, we assign two jobs to the epsilon operator. First, an order is projected upon the set of elements that fall under the description. Second, the first element from this well-ordered set is chosen. Both roles of the choice operator will herein be made dependent upon the situation. We illustrate the situation by the following example:

(36) The island on Lake Constance is nice.
a. N εx [island on Lake Constance(x)]

Example (36a) is the representation of (36). The property *island on Lake Constance* is common to three objects: Mainau, Reichenau, and Lindau. The description εx *[island on Lake Constance(x)]* denotes the first element of the set of islands on Lake Constance. This expression might denote different islands according to different situations. If we hear sentence (36) from a Reichenau fisherman, he probably means the island Reichenau; if we encounter the same sentence during a guided tour through Lindau it will rather be the island Lindau that is meant; however, uttered by the Earl, owner and occasional inhabitant of Mainau, the sentence is sure to be about the island Mainau. We can assign one indexed epsilon operator to each of these situations:

(37) $\boldsymbol{\varepsilon}_{\text{fisherman}}x$ [island on Lake Constance(x)]: Reichenau
$\boldsymbol{\varepsilon}_{\text{tourist guide}}x$ [island on Lake Constance(x)]: Lindau
$\boldsymbol{\varepsilon}_{\text{earl}}x$ [island on Lake Constance(x)]: Mainau

A description of the form $\varepsilon_i x$ Fx denotes that individual with the property F that is chosen first in a certain situation i. In (37) we have named this individual. A more general description of the difference between these situation-dependent choice functions is to state the well-ordering that they impose on a set. This is shown in (38), where the islands are abbreviated by their initials.

(38) $\boldsymbol{\varepsilon}_{\text{fisherman}}x$ [island(x)]: the first element from {L, M, R} with the order: (R > L > M)
$\boldsymbol{\varepsilon}_{\text{tourist guide}}x$ [island(x)]: the first element from {L, M, R} with the order: (L > M > R)
$\boldsymbol{\varepsilon}_{\text{earl}}x$ [island(x)]: the first element from {L, M, R} with the order: (M > R > L)

If the fisherman utters sentence (36) by the description *the island on Lake Constance* he means Reichenau. Using the stated order, we can go further and analyze the following sentence (39), which is supposed to be uttered by the fisherman next. The description *the other island* is represented as in (40):

(39) The other island is nice, too.
(40) $\varepsilon_{\text{fisherman}}y$ [island(y) & $y \neq \varepsilon_{\text{fisherman}}x$ [island(x)]]

The description $\varepsilon_{\text{fisherman}}x$ [island (x)] denotes the island chosen first, which in the fisherman's situation is Reichenau. The description $\varepsilon_{\text{fisherman}}y$ [island(y) & $y \neq \varepsilon_{\text{fisherman}}x$ [island(x)]] denotes the island chosen second (viz. the chosen island that is not the one chosen first), that is, the island Lindau.

2.5 *The Rules of Thematization, Rhematization, and Reduction*

In this section we will present rules allowing us to transform sentences with descriptions and to identify descriptions with one another. In a given situation a description always denotes exactly one individual. The content of the common noun phrase within the description, however, may apply to several individuals. The more individuals meet the description, the more contextual information is needed to identify the intended individual. If the description is true of only a few individuals, it is easier to identify the right one. If I talk about *the city*, I am less specific than if I am talking about *the city on Lake Constance*, *the city with a university*, or even *the city on Lake Constance with a university*. In the most unambiguous phrase there is only one individual left to meet the description and thus to be denoted by it. This is the Russell-type case, which is context independent. We can view E-type pronouns as being the other extreme, as they seem to have no restriction by a common noun phrase at all and are strongly context dependent (cf. section 3).

According to the maxim of exactness (cf. section 1.6), the description will be further specified if its descriptive content is true for many individuals, until only one individual falls under the descriptive content. One possibility for specification will be formulated in our rules. Here the predicate that is *applied to* a description shall be *included in* it. We call this the extension of the theme. To this procedure correspond the transition from (41) to (42) and the identification of the two descriptions in (43):

(41) The island is nice.
(42) The island that is nice is nice.
(43) the island = the island that is nice

We also call this thematization of the rheme. In this context the description will be called the *theme*, and what is said about it, that is, the predicate of the statement, will be called the *rheme*. The sentence that is relevant is always the smallest construction that contains the description and that can be assigned a truth-value (this is roughly parallel to Heim's conception of a *minimal sentence*). Syntactically, the property added by the integration of the predicate into the description is usually realized as a relative clause.

(44) Thematization of the rheme (extension of the theme)
For every situation i with G $\varepsilon_i x$ Fx there is a situation j, such that:
$\varepsilon_i x\ Fx = \varepsilon_j x\ Fx = \varepsilon_j x\ [Fx\ \&\ Gx]$
(and thus G $\varepsilon_j x$ [Fx & Gx] after the rule of identity by Leibniz)

As the two descriptions have different common noun phrases (i.e., different descriptive contents), the choice function Φ_i will in general assign two different individuals to the two different sets. However, there is at least one choice function Φ_j which assigns the same individual to both sets. If the description is unambiguous, then the Russell-type description coincides with the Hilbert-type description:

(45) For each situation i with G $\varepsilon_i x$ Fx,
$\varepsilon_i x\ [Gx\ \&\ Fx] = \varepsilon_i x\ Fx = \iota x\ Fx$ is valid
if there is exactly one individual a with the property F.

We now can describe our examples (41)-(43) more closely. Example (41a) represents the initial statement in a situation i, which is supposed to be the current context of conversation. We apply the rule of thematization to get (42a). Finally, there is a situation j, in which both descriptions denote the same individual. This situation j becomes our new context of conversation:

(41) a.	The island is nice.	N $\varepsilon_i x$ Ix
(42) a.	The island that is nice is nice.	N $\varepsilon_j x$ [Ix & Nx]
(43) a.	the island = the island that is nice	$\varepsilon_j x\ Ix = \varepsilon_j x\ [Ix\ \&\ Nx]$

The empty set does not cause any problems. If F is an empty property, then $\varepsilon_i x$ Fx denotes an arbitrary element with the property G. In that case (Fx & Gx) is an empty set as well, and $\varepsilon_i x$ [Fx & Gx] denotes the same arbitrary element as $\varepsilon_i x$ Fx. Thus, we can form (46a) on the basis of (46) and identify both descriptions in (46b):

(46)	The ghost rumbles.	R $\varepsilon_i x$ Gx
a.	The ghost that rumbles rumbles.	R $\varepsilon_j x$ [Gx & Rx]
b.	the ghost = the ghost that rumbles	$\varepsilon_j x\ Gx = \varepsilon_j x\ [Gx\ \&\ Rx]$

In (47), we repeat the extension of the theme (44) and show intuitively that this rule is valid. We may distinguish three cases (47i-iii):

(47) Thematization of the rheme (extension of the theme)
For every situation i with G $\varepsilon_i x$ Fx there is a situation j, such that:
$\varepsilon_i x\ Fx = \varepsilon_j x\ Fx = \varepsilon_j x\ [Fx\ \&\ Gx]$
(i) The property F in the description $\varepsilon_i x$ Fx is empty, and thus *a fortiori* the property (Fx & Gx) of $\varepsilon_j x$ [Fx & Gx] is empty as well. Then in every situation both descriptions denote the same arbitrary element.
(ii) The property in the description $\varepsilon_i x$ Fx denotes exactly one element. In all situations this element is identical with the element denoted by $\varepsilon_j x$ [Fx & Gx].
(iii) More than one individual fall under the property F. As $\varepsilon_j x$ [Fx & Gx] has the property F, there is at least one situation j in which the two descriptions denote the same individual: $\varepsilon_j x\ Fx = \varepsilon_j x\ [Fx\ \&\ Gx]$.

There are situations where we can reduce the descriptive content within the description. This we will call *reduction of the theme.* Also, we have another reversal of our techniques, the rhematization of the theme, which is just one particular form of the Hilbert rules already introduced:

(48) Rhematization of the theme (= Hilbert's rule for the epsilon operator):
For all situations i, if $\exists x\ Fx$, then $F\ \varepsilon_i x\ Fx$

We can see from this rule how single properties, as parts of complex properties, can be attributed to the whole by simplification of conjunctions. In the light of this view of Hilbert's rule as amounting to a rule of thematization, this rule is justified by linguistic analysis and can be seen as intuitively valid for descriptions of everyday language. Some sentences may be used to illustrate the situation:

(49)	The nice island is famous.	$F\ \varepsilon_i x\ [Ix\ \&\ Nx]$
a.	The nice island is an island.	$I\ \varepsilon_i x\ [Ix\ \&\ Nx]$
b.	The nice island is nice.	$N\ \varepsilon_i x\ [Ix\ \&\ Nx]$
c.	The island is nice.	$N\ \varepsilon_j x\ Ix$
d.	the island = the nice island	$\varepsilon_j x\ Ix = \varepsilon_j x\ [Ix\ \&\ Nx]$

We get (49a) and (49b) by rhematization or by the first Hilbert rule. Their conjunction is equivalent to $\exists x\ [Ix\ \&\ Nx]$. The predication in (49c) can be justified by identity laws and (49d), which in turn is obtained by the rule of thematization.

We also have a rule for reducing the common noun phrase within a description, demonstrated under (50).

(50) Reduction of the theme
For every situation i with $F\ \varepsilon_i x\ [Fx\ \&\ Gx]$ and $G\ \varepsilon_i x\ [Fx\ \&\ Gx]$
there is a situation j where $\varepsilon_j x\ [Fx\ \&\ Gx] = \varepsilon_j x\ Gx$ and $F\ \varepsilon_j x\ Gx$

If we want to apply the reduction rule to (51), we first get (51a) and (51b) by applications of the rhematization rule, which are the appropriate input for the reduction rule that now leads to (51c). The result can be inserted in (51) and finally gives the reduced version (51d), for a situation where we can use this simple description to refer to the intended individual:

(51)	The city on Lake Constance has a university.	$H(\varepsilon_i x\ [Cx\ \&\ On(x, lc)], u)$
a.	The city on Lake Constance is a city.	$C(\varepsilon_i x\ [Cx\ \&\ On(x, lc)])$
b.	The city on Lake Constance is on Lake Constance.	$On(\varepsilon_i x\ [Cx\ \&\ On(x, lc)], lc)$
c.	the city on Lake Constance = the city	$\varepsilon_j x\ [Cx\ \&\ On(x, lc)] = \varepsilon_j x\ Cx$
d.	The city has a university.	$H(\varepsilon_j x\ Cx, u)$

We have introduced the three rules of thematization, of rhematization, and of reduction of the theme, which are well motivated with respect to natural language use. In the next section we will analyze difficult sentences with descriptions which have posed problems much discussed in the research literature.

3. *E-type Pronouns and Epsilon Terms*

Our theory of descriptions, which constitutes an extended version of the classical theory of Russell, will now be applied to a special problem, namely, the description of anaphoric pronouns. According to Evans (1977), certain anaphoric pronouns can be analyzed as definite descriptions. He calls these pronouns E-type pronouns. Heim (1982:43) has commented on this analysis:

> Then there was Evans' proposal to analyze the problematic anaphoric pronouns as paraphrases of certain definite descriptions that can be constructed from the antecedent and the sentence it occurs in in a systematic way. This proposal avoids both the objections against Geach and against the Griceans. However, it has little substance if the proper analysis of (anaphoric) definite descriptions remains itself unclear, and it is subject to criticism if coupled with the standard analysis of definite descriptions that Evans presupposes. The problem is that definite descriptions under the standard analysis carry an implication that their referent satisfies the descriptive predicate uniquely, and it is not true, or at the very least doubtful, that this uniqueness is required for an anaphoric use of pronoun or definite description to be felicitous.

Despite this objection, the E-type analysis became well established and is now commonly used for those anaphoric pronouns. The uniqueness assumption, however, remains what Heim (1990:142) has aptly called the Achilles' heel of the E-type analysis of pronouns. Using our analysis of descriptions, where Russell's iota operator is replaced by a family of Hilbert-type epsilon operators, we would like to make even this part of the theory invulnerable. In this application, our extended theory of descriptions will prove suitable for the analysis of special problems of the semantics of anaphora in a way that strongly supports our analysis.

3.1 *E-type Pronouns*

E-type pronouns are distinguished from other pronouns which can be represented as bound variables in the following way: although their antecedent is a quantifier phrase, they themselves stand outside the scope of this quantifier phrase. Thus, (52) is represented in a standard format as (52a).

(52) A man comes. He whistles.
a. $\exists x\ (Mx\ \&\ Cx\ \&\ Wx)$

However, this representation is questionable, as it violates the principle of compositionality and allows the quantifier's scope to extend beyond its syntactic scope. Evans therefore provides for sentence (52) the paraphrase (52b), where he represents the anaphoric pronoun as a definite description that describes the antecedent. He formalizes the description by using the iota operator, as in (52c).

(52) b. A man comes. The (**unique**) man that comes whistles.
c. $\exists x\ (Mx\ \&\ Cx)\ \&\ W\ \iota x\ (Mx\ \&\ Cx)$

Evans identifies the pronoun with the antecedent. We shall call this procedure *Evans-identification*, though we have to admit that the semantic conditions of this identification are not well understood. Perhaps it has to be a syntactic process.

Thus, Evans has solved the problems of the classical analysis mentioned above: because the sentence $W\ \iota x\ [Mx\ \&\ Cx]$ can be assigned a truth-value, compositionality can be maintained. The quantifier scope does not extend beyond the sentence, and the interpretation (52a) that has turned out to be too strong cannot be derived.

An E-type pronoun can be seen as a completely unspecified description which is given a descriptive content only by the context. According to Evans, the *descriptive material* (i.e., the descriptive content in our terminology) is provided not merely by the sentence containing the antecedent, but also "by the material which the speaker supplies upon demand" (Evans 1977:517). In order to make the processes more precise, we will use the rule of thematization of the rheme introduced in section 2.5, which is to be compared with Evans' rule (1977:535) or with the transformations proposed by Heim (1990: 170) and which has the advantage of being of a purely semantic nature.

Here we can use Evans-identification. We can conceive of an E-type pronoun as an epsilon term which does not contain any descriptive content

and just means something like *the individual mentioned first* (Egli 1991:21). After the rule of thematization, it can be identified with other epsilon terms with which it is anaphorically related.

Let us consider example (52) (repeated as (53)) from this point of view. The formalization is given directly after the respective natural language expression. To begin with, we have to state the referential identity of the antecedent with the pronoun, as shown in (53a). Once this relationship has been established, as in (53b), let the term then be substituted for the pronoun, as in (53c).

(53)	A man comes. He whistles.	$C\ \varepsilon_i x\ Mx\ \&\ Wa$
a.	[A man]$_1$ comes. He$_1$ whistles.	
b.	[a man]$_1$ = he$_1$	$\varepsilon_i x\ Mx = a$
c.	A man comes. The man whistles.	$C\ \varepsilon_i x\ Mx\ \&\ W\ \varepsilon_i x\ Mx$

This is the simplest case. However, we may as well specify the descriptive content within the description, which can be done by the rule of thematization of the rheme. We then can get (53d) from (53) and proceed with the following steps as we did above.

(53) d.	[A man who comes]$_1$ comes. He$_1$ whistles.	
e.	[a man who comes]$_1$ = he$_1$	$\varepsilon_i x\ [Mx\ \&\ Cx] = a$
f.	The man who comes whistles.	$W\ \varepsilon_i x\ [Mx\ \&\ Cx]$

Equipped with this apparatus, we are able to describe even sentences with more than one individual falling under the descriptive content in the description. We here repeat (20b) as (54). Considering the semantics of *other*, we can represent (54) as (54a), where *r* is a constant for *the room*:

(54) A man enters the room. An*other* man leaves the room.
a. $E\ (\varepsilon_i x\ Mx,\ r)\ \&\ L\ (\varepsilon_i y\ [My\ \&\ y \neq \varepsilon_i x\ Mx],\ r)$

In our situation i there is obviously more than one man. While the one mentioned first is entering the room, the one mentioned second leaves.

3.2 *Bishop Sentences*

Using the rules discussed so far, we can now try to solve some puzzles connected with bishop sentences, which in most analyses cause difficulties because Russell-type descriptions involve the presupposition of uniqueness, which they seem to violate. We shall try to solve the problems with the help of our theory of ordinal numbers. Though we are aware of the difficulties of representing the conditional by material implication, we shall do so in order to simplify our treatment.

(55) If one linguist meets another, he nods to him.
a. If a first linguist meets a second, he nods to him.
b. the first linguist: $\varepsilon_i x\ Lx$
c. the second linguist = the first linguist who is not identical with the first linguist: $\varepsilon_i y\ [Ly\ \&\ y \neq \varepsilon_i x\ Lx]$
d. [the first linguist]$_1$ = he$_1$ $\quad \varepsilon_i x\ Lx = a_1$
e. [the second linguist]$_2$ = he$_2$ $\quad \varepsilon_i y\ [Ly\ \&\ y \neq \varepsilon_i x\ Lx] = a_2$
f. $M(a_1, a_2) \rightarrow N(a_1, a_2)$

We now want to show that our formalization can be shown to be equivalent to a standard formalization by application of our rules. We consider the converse of (55f) under (56a). Throughout the argument we presuppose that there are linguists. Then we apply the Rules of thematization or rhematization in order to get a form which we may transform by the first Hilbert rule into (56b).

(56) a. $M(a_1, a_2)\ \&\ \neg N(a_1, a_2)$
b. $\exists x\ \exists y\ [Lx\ \&\ Ly\ \&\ x \neq y\ \&\ M(x, y)\ \&\ \neg N(x, y)]$

According to our rules of rhematization and thematization, a_2 above is identical to a_3 in (57a). The sentence (56a) is transformed into (57b).

(57) a. $\varepsilon_j y\ [La_1\ \&\ Ly\ \&\ y \neq a_1\ \&\ M(a_1, y)\ \&\ \neg N(a_1, y)] = a_3$
b. $[La_1\ \&\ La_3\ \&\ a_3 \neq a_1\ \&\ M(a_1, a_3)\ \&\ \neg N(a_1, a_3)]$

According to Hilbert's rule, this is equivalent to (57c).

(57) c. $\exists y\ [La_1\ \&\ Ly\ \&\ y \neq a_1\ \&\ M(a_1, y)\ \&\ \neg N(a_1, y)]$

By thematization of the rheme we get the identity of a_1 with a_4 in (57d), and from (57c) we derive the new form (57e).

(57) d. $\varepsilon_k x\ \exists y\ [Lx\ \&\ Ly\ \&\ y \neq x\ \&\ M(x, y)\ \&\ \neg N(x, y)] = a_4$
e. $\exists y\ [La_4\ \&\ Ly\ \&\ y \neq a_4\ \&\ M(a_4, y)\ \&\ \neg N(a_4, y)]$

According to the first Hilbert rule this is equivalent to (56b), which we wanted to get.

(56) b. $\exists x\ \exists y\ [Lx\ \&\ Ly\ \&\ x \neq y\ \&\ M(x, y)\ \&\ \neg N(x, y)]$

(56a) follows from (56b) provided there are linguists. The opposite direction of the equivalence is obtained unproblematically by the reduction of the theme and by the simplification of conjunctions. As a result of this proof, we obtain (58) as a theorem of our system. If we negate both sides of the

equivalence, we get the equivalence of (57c) with (56).

(58) If $\exists x\ Lx$, then (56b) is equivalent to (56a).
(59) $\forall x \forall y\ [(Lx\ \&\ Ly\ \&\ x \neq y\ \&\ M(x, y)) \rightarrow N(x, y)]$

3.3 *Donkey Sentences, Sageplant Sentences and the Proportion Problem*

The same technique suffices to demonstrate the equivalence of donkey sentences, which are mere conditionals, with our logical form for these sentences (Egli 1991). Sageplant sentences like (60) present little difficulty, since we have a semantic representation for *the first barn* and *the second barn* which lends itself to the analysis of these sentences because it does not involve any presupposition of uniqueness.

(60) If a farmer builds a barn, he often builds another that is connected with the first by a path.

In order to solve the proportion problem with these rules, we would have to add a theory of plurals. We think that the rules could then be adapted to provide a solution to these problems. But this must be the topic of another paper.

4. ***Summary***

1. We presuppose a syntactic theory compatible with the present semantic views but do not treat syntax here.
2. We use a family of choice functions and their representation by a parametrized epsilon operator as semantic representation for *a* and *the*. The logical form of a sentence is very near to the surface form.
3. We present a theory of ordinal numbers. These are used for the linguistic selection of a unique individual by description, on a par with techniques of supplementing the common noun phrases of the descriptions.
4. E-type pronouns are reconstructed with the help of our modified epsilon operator, not with the help of Russell's iota operator. Pronouns are reconstructed as constituting expressions like *the individual, the first individual*, and the like.
5. Additional rules besides Hilbert's rules are imposed on the epsilon operator, like the rules of thematization of the rheme, the rhematization of the

theme, and the reduction of the theme. The rule of thematization furnishes a linguistic motivation of Hilbert's rule, with which it is equivalent.

6. Invariably one needs the principle that every set can be well-ordered, if one is to construct a theory of ordinal numbers. Ockham's razor seems to say that because we can define a suitable semantic theory of the definite article from this well-ordering, we should adopt this definition without postulating a Russellian iota operator besides.

Notes

1. Cf. Hilbert & Bernays (1939:12): "Das ε-Symbol bildet somit eine Art Verallgemeinerung des μ-Symbols [which is derived from the iota operator] für einen beliebigen Individuenbereich. Der Form nach stellt es eine Funktion eines variablen Prädikats dar, welches außer demjenigen Argument, auf welches sich die zu dem ε-Symbol gehörige gebundene Variable bezieht, noch freie Variablen als Argumente ("Parameter") enthalten kann."

2. We write εx Fx, where the x directly following the ε specifies the variable that is bound in the open sentence (here Fx). Hilbert and Bernays index the ε with x: ε_x. We will use the non-indexed form, analogous to the convention for quantifiers. The scope of the epsilon operator will be given in parentheses only when necessary. In the following we will sometimes use the term "set" instead of "property" to make the presentation clearer. Actually, the transition from a property (characteristic function) to the terminology of set theory needs to be justified in each case; however, to do so here would be rather long-winded.

3. A semantic interpretation, however, has to satisfy the following three syntactic conditions (Leisenring 1969:33):
   (i) (ε_0) $\exists$x Fx $\rightarrow$ F εx Fx must be valid.
   (ii) Every term of the form εx Fx must get a value because, according to Hilbert, it is permissible to substitute free variables by arbitrary ε-terms.
   (iii) $\forall$x (Px $\leftrightarrow$ Qx) $\rightarrow$ (εx Px = εx Qx) must be valid.

4. Asser (1957:33): "Das Zeichen ε schließlich ist eine Variable für Auswahlfunktionen des Individuenbereichs J. Dabei ist eine Auswahlfunktion von J eine Abbildung Φ, welcher jeder nicht-leeren Teilmenge von J ein eindeutig bestimmtes Element dieser Teilmenge zuordnet." There is a problem regarding the empty set. Asser gives four possibilities for assigning an element to an empty set:
   (i) εx Fx denotes an arbitrary (but fixed) individual (= first suggestion by Asser).
   (ii) Φ is not defined for the empty set (= second suggestion by Asser).
   (iii) Φ assigns the same individual to the empty set as to the universe of discourse: $\Phi(\varnothing) = \Phi(U)$ (suggestion after Hermes 1965; this is also mentioned by Asser, who states it as a consequence of the definition of the ε-symbol by the η-symbol as proposed by Hilbert and Bernays).
   (iv) Finally, Asser makes a fourth suggestion (the third in the above-mentioned article). This, however, is very complicated. "Allerdings ist dieser Begriff von Auswahlfunktion so kompliziert, daß sich seine Verwendung in der inhaltlichen Mathematik kaum empfiehlt." (Asser 1957:57).
   Following Leisenring we will stay with the solution given in (i).

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