Necessity Modals, Disjunctions, and Collectivity

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The Problem and Solution. Standard semantics for necessity modals and disjunction (e.g. Kratzer (1991), Partee and Rooth (1983)) validate the intuitively invalid *Ross inference*: *Must(p), therefore Must(p or q)*. Solutions to this problem usually rest on *Diversity*: the thesis that *Must(p or q)* conveys that both p and q are compatible with the relevant set of worlds (see, e.g. Simons (2005a); von Fintel (2012)). I argue that Diversity is not strong enough to explain the full range of data. First, as Sayre-McCord (1986) and Fusco (2015) argue in the case of 'ought', adding premises that entail Diversity does not make a Ross inference seem valid (i.e. *May(p), May(q), Must(p), therefore Must(p or q)* still seems invalid). Second, analyzing why these inferences are invalid leads to an illuminating piece of data: sentences of the form *Must(p or q)* license the inference to indicative conditionals of the form *if* $\neg p$, *Must(q)* and *if* $\neg q$, *Must(p)* (see also Fusco (2015)). Generating these predictions semantically means that over-and-above the standard truth conditions of *Must(p or q)*, modals with disjunctive prejacents convey something stronger than the Diversity analysis predicts, namely what I call *Independence*: that *p-and-not-q*, on the one hand, and *q-and-not-p*, on the other, are compatible with the set of worlds quantified over by *Must*. I show that this is formally equivalent to the following requirement: *Must(p or q)* is true at w iff $\{[\![p]\!], [\![q]\!]\}$ is a **minimal cover** of the relevant set of worlds $R(w)$ (where $[\![p]\!]$ is the set of worlds where p is true, and $\{[\![p]\!], [\![q]\!]\}$ is a minimal cover of a set S iff $[\![p]\!] \cup [\![q]\!]$ is a superset of S and no proper subset of $\{[\![p]\!], [\![q]\!]\}$ is a superset of S).[1]

Framework for Implementation. I model these ideas using a *bilateral* version of propositional *inquisitive semantics*. I use *inquisitive semantics* because it allows the semantics of a modal to be sensitive to the disjunctive structure of its prejacent. I use a *bilateral* version because it allows us to retain *duality* between possibility and necessity modals. Given a countable set of atomic sentence letters, At, and $p \in \mathsf{At}$, wffs are defined by the following grammar: $p \mid \neg\phi \mid \phi \vee \psi \mid \Box\phi$.

A **model** $\mathcal{M}$ is a triple $\mathcal{M} = (W_{\mathcal{M}}, R_{\mathcal{M}}, V_{\mathcal{M}})$ where $W_{\mathcal{M}}$ is a set of worlds, $R_{\mathcal{M}}$ is a function from worlds to sets of worlds ($R_{\mathcal{M}} : W \mapsto \wp(W)$), and $V_{\mathcal{M}}$ is a function from atomic sentences to truth sets ($V_{\mathcal{M}} : \mathsf{At} \mapsto \wp(W)$). A **bilateral proposition** P in a model $\mathcal{M}$ is a pair (P^+, P^-) of downward-closed (w.r.t. the subset relation) sets of sets of worlds, such that their intersection is the singleton containing the empty set. I use the following notions familiar from inquisitive semantics (where P is a bilateral proposition):

$$\mathsf{info}(P^+) = \bigcup P^+ \quad \text{(the set of worlds where } P \text{ is true)}$$
$$\mathsf{alt}(P^+) = \{s \in P^+ \mid \neg\exists t \in P^+ : \ t \supset s\} \quad \text{(the positive alternatives offered by } P)$$
$$\mathsf{alt}(P^-) = \{s \in P^- \mid \neg\exists t \in P^- : \ t \supset s\} \quad \text{(the negative alternatives offered by } P)$$

To these I add the notion of a minimal cover:

$$C \text{ is a } \textbf{cover} \text{ of } S \text{ iff } S \subseteq \bigcup C$$
$$C \text{ is a } \textbf{minimal cover} \text{ of } S \text{ iff } C \text{ is a cover of } S \text{ and there is no } C' \subset C : C' \text{ is a cover of } S$$

[1]Compare the equivalence between Diversity and the notion of a *supercover* found in Simons (2005a). Precedents for a stronger requirement like Independence include Menéndez Benito (2005, 2010); Aloni and Ciardelli (2013).

Minimal Covering Semantics. I write $[\phi] = ([\phi]^+, [\phi]^-)$ for the bilateral proposition expressed by ϕ.[2]

$$
\begin{aligned}
[p] &= (\wp(V(p)), \wp(W \backslash V(p))) \quad \text{when } p \in \mathsf{At} \\
[\neg\phi] &= ([\phi]^-, [\phi]^+) \\
[\phi \vee \psi] &= ([\phi]^+ \cup [\psi]^+, [\phi]^- \cap [\psi]^-) \\
[\Box\phi]^+ &= \wp(\{w \in W \mid \mathsf{alt}([\phi]^+) \text{ is a minimal cover of } R(w)\}) \\
[\Box\phi]^- &= \wp(\{w \in W \mid \text{ there is a non-empty } R' \subseteq R(w) \text{ such that} \\
&\qquad \mathsf{alt}([\phi]^-) \text{ is a minimal cover of } R'\}
\end{aligned}
$$

I treat $\phi \wedge \psi$ as an abbreviation of $\neg(\neg\phi \vee \neg\psi)$ and $\Diamond\phi$ as an abbreviation for $\neg\Box\neg\phi$.

Entailment. ϕ entails ψ in a model $\mathcal{M}$ (written $\phi \vDash_{\mathcal{M}} \psi$), when $\mathsf{info}^+([\phi]_{\mathcal{M}}) \subseteq \mathsf{info}^+([\psi]_{\mathcal{M}})$.

Results. In any model $\mathcal{M}$ where $p \vee q$ obeys Hurford's constraint (i.e. $V_{\mathcal{M}}(p) \nsubseteq V_{\mathcal{M}}(q)$ and $V_{\mathcal{M}}(q) \nsubseteq V_{\mathcal{M}}(p)$), we have the following package of desirable results:

$$\Box(p \vee q) \vDash_{\mathcal{M}} \Diamond(p \wedge \neg q) \qquad \text{(Independence)}$$

$$\Box p \nvDash_{\mathcal{M}} \Box(p \vee q) \qquad \text{(Ross Inference)}$$

$$\Diamond(p \vee q) \vDash_{\mathcal{M}} \Diamond p \qquad \text{(Free Choice)}$$

$$\neg\Diamond(p \vee q) \vDash_{\mathcal{M}} \neg\Diamond p \qquad \text{(Impossibility Distribution)}$$

$$\neg\Box(p \vee q) \vDash_{\mathcal{M}} \neg\Box p \qquad \text{(Unnecessity Distribution)}$$

Collectivity. This attractive package of results comes at the price of generating truth value gaps for sentences containing modals. For example, $\Diamond(p \vee q)$ is true only if p and q are both possible, while it is false only if both are impossible. If only one or the other is possible, $\Diamond(p \vee q)$ is neither true nor false. So, my theory predicts that possibility modals behave like distributive, *homogeneous* predicates of plurals (this analogy between possibility modals and plural predicates has been noticed by Simons (2005b) and Goldstein (2019)).

My minimal covering semantics for necessity modals opens up an unnoticed aspect of the analogy between plural predicates and nominal terms, on the one hand, and the interaction between disjunctions and modals, on the other: necessity modals behave like *collective predicates* of disjunctions. For a summary, my semantics account of the interaction between necessity modals and disjunctions derives the following properties commonly associated with collective predicates like 'performed' (Dowty, 1987; Lasersohn, 1990; Križ, 2015):

Nondistributivity	Al and Bo performed *Happy Days* $\nRightarrow$ Al performed *Happy Days* You must sell or destroy the piano $\nRightarrow$ You must sell the piano
Involvement	Al and Bo performed *Happy Days* $\Rightarrow$ Al helped perform *Happy Days*. You must sell or destroy the piano $\Rightarrow$ Selling the piano is one way to do what you must.
Upward Failure	Al performed *Happy Days* $\nRightarrow$ Al and Bo performed *Happy Days* You must sell the piano $\nRightarrow$ You must sell or destroy the piano
Homogeneity	Al and Bo didn't perform *Happy Days* $\Rightarrow$ Al didn't perform *Happy Days* You don't have to sell the piano or destroy it.$\Rightarrow$ You don't have to sell the piano.

By drawing out this parallel, I hope to further the case that the puzzling logical behavior that disjunction gives rise to when under the scope of modal operators can be assimilated to the theory of plurals in the nominal domain.

[2]See Groenendijk and Roelofsen (2010), Aher (2012), Willer (2018), and Aloni (2018) for other uses of bilateral inquisitive semantics.

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