Odds, Probabilities, Scores, and the Interpretation of Measure Phrases

It's natural to suppose that purely numerical measure expressions, such as *three* or *two thirds*, name degrees on a single numerical scale. Our aim is to examine a broader class of measure phrases— including *two in three* and *six to one*—that demonstrates that even in the absence of unit names, apparently purely mathematical measure phrases refer to degrees across a number of distinct and incommensurable scales. Indeed, some of them—such as sports scores—show that measure phrases can actually refer irreducibly to tuples of degrees. Both of these moves require broadening the usual conception of what degrees are and of what measure phrases can mean.

Proportional measure phrases can be expressed in a number of syntactic forms. They are not interchangeable even when they name arithmetically identical fractions:

(1) a. Floyd is {33% | one third | #1 in 3 | #1 to 2 | #.33} as tall as Clyde.
b. Let's disburse {33% | one third | 1 in 3 | #1 to 2 | #.33} of the donations.
c. Her odds of winning are {33% | #?one third | 1 in 3 | 1 to 2 | .33}.
d. The probability of winning is {33% | ??one third | 1 in 3 | #1 to 2 | .33}.
e. I think {??33% | one third | #1 in 3 | #1 to 2 | .33} is a small number.

Mathematically, all of these measure phrases are, of course, (nearly) equivalent, all representing the proportion $\frac{1}{3}$. But the variation in their acceptability in (1) demonstrates that linguistically, they can't name the same degree. The issue, we will argue, is that despite the absence of a overt unit name like 'feet', these measure phrases pick out fractional degrees on different scales. By definition, degrees on the same scale are totally ordered, and so can freely be compared. Degrees on different scales cannot be, modulo complications (von Stechow 1984, Kennedy 1997; cf. Bale 2006). Thus explicit comparisons such as (2) can test for scale-mate degrees:

(2) a. 2 is greater than {#?33% | one third | #1 in 3 | #1 to 2 | .33}.
b. 90% is greater than {33% | one third | #1 in 3 | #1 to 2 | #?.33}.
c. 2 in 3 is {greater | better odds} than {??33% | #?one third | 1 in 3 | 1 to 2 | #?.33}.
d. 0.9 is a higher probability than {?33% | #?one third | 1 in 3 | 1 to 2 | .33}.

These incommensurability contrasts demonstrate that there are several distinct scales at play here: purely numerical, percentage-based, odds, and probabilities. Sports scores represent yet another form of measure phrase:

(3) a. Colombia beat Uruguay ({with a score of | by}) six to four.
b. Colombia beat Uruguay *(by) one.

Despite a syntactic resemblance, the measure phrase in (3a) isn't proportional—and crucially, it supports entailments to differential measure phrases, as in (3b). Multidimensional measurement (e.g. *an area of 3in by 2in*) has similar properties. In both these sets of cases, multiplying both figures by a constant does not yield an equivalent measure phrase: a 6–4 score cannot be 'reduced' to an equivalent 3–2 one.

Numerals are often treated as degrees (Hackl 2000, Geurts & Nouwen 2007 a.o; cf. Moltmann 2017, Snyder 2017). Fractional measures are less studied, but conceptualized in similar terms (Ahn & Sauerland (2015, 2017), Solt (2018)). None of these approaches distinguish between the superficially purely numerical scales, as we must here. To our knowledge, the various unusual syntactic shapes of measure phrases at issue here also awaits an explicit semantics.

The first analytical challenge is the proportional measure phrases. Although there are interesting compositional differences among them (and a revealing resemblance to the grammar of arithmetic, including e.g. operators like *plus*; Gobeski 2019), the essential task theoretically is ensuring that they measure along different scales:

(4) a. $[\![\, \textit{to}_{\textit{degree}} \,]\!] = \lambda d \lambda d' \,.\, (d/d+d')_{\in S_{\textbf{odds}}}$
b. $[\![\, \textit{in}_{\textit{degree}} \,]\!] = \lambda d \lambda d' \,.\, (d/d')_{\in S_{\textbf{probability}}}$

(5) a. $[\![\, \textit{1 to}_{\textit{degree}}\ \textit{2} \,]\!] = (1/3)_{\in S_{\textbf{odds}}}$
b. $[\![\, \textit{1 in}_{\textit{degree}}\ \textit{3} \,]\!] = (1/3)_{\in S_{\textbf{probability}}}$

These fit in differently in different contexts, of course, but in general, in *probability of 1 in 3*, we take the noun to presuppose that its complement is on the probability scale, and corresponding assumptions extend it to predicative contexts like (1). For proportional measurement in equatives as in (1a), see Gobeski & Morzycki (2018). As for scores, they don't express proportions. Instead, we suggest that they denote tuples of degrees, and embedding verbs like *beat* must take them as arguments:

(6) a. $[\![\, \textit{to}_{\textit{score}} \,]\!] = \lambda d \lambda d' \,.\, \langle d', d \rangle$ b. $[\![\, \textit{beat} \,]\!] = \lambda D_{d \times d}\, \lambda x \lambda y \,.\, \textbf{beat}(y, x, D)$
b. $[\![\, \textit{Colombia beat Uruguay 6 to 4} \,]\!] = \textbf{beat}(\textbf{Colombia}, \textbf{Uruguay}, \langle 6, 4 \rangle)$

Differential uses are more complicated: a meaningful differential *by* maps a degree-pair to a degree generalized quantifier, which then QRs (**diff** is a difference function):

(7) a. $[\![\, \textit{by}_{\textit{differential}} \,]\!] = \lambda d \lambda P_{\langle d \times d, t \rangle} \,.\, \exists D[P(D) \wedge \textbf{diff}(D) = d]$
b. Brazil beat Uruguay by$_{\text{differential}}$ 2.
$[\![\, \textit{by}_{\textit{differential}}\ \textit{2} \,]\!]\, ([\![\, \lambda D\ \textit{Brazil beat Uruguay}\ t_D \,]\!])$
$= \exists D_{d \times d}[[\![\, \lambda D\ \textit{Brazil beat Uruguay}\ t_D \,]\!]\,(D) \wedge \textbf{diff}(D) = 2]$
$= \exists D_{d \times d}[\textbf{beat}(y, x, D) \wedge \textbf{diff}(D) = 2]$

This correctly predicts scope ambiguities with differentials (e.g. *Colombia must beat Uruguay by 2*) but not elsewhere. It also predicts the obligatoriness of *by* in its differential use. In principle, one could also have made explicit use of distinct measure functions here, but the simpler representation in (7) suffices. Distinguishing among scales in this way accounts for the incommensurability facts without major theoretical innovations, except for one insight: the odds scale is a proper subscale of the probability scale, because all odds measure phrases can also occur in probability contexts (and not vice versa).

The innovations proposed here extend to various other numerical expressions, such as areas, volumes, coordinates, and specialized uses such as aspect ratios (e.g., *16:9* or *4:3*; **4/3*). Certain expressions in this family interact with modals, and suggest connections to the increasingly ample literature on modified numerals. More broadly, this represents a step toward relating basic degree semantics to the specialized subgrammars of particular professional/intellectual milieus.

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