The left-CONS2 Constraint
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Introduction: A well-known claim about natural language (NL) determiners is that they obey the Conservativity Constraint (Barwise & Cooper, 1981; Keenan & Stavi, 1986), which implies that NL-determiners denote CONS1 functions (i.e., \( DPQ \Leftrightarrow DPP \cap Q \)). One might wonder whether NL-determiners denote CONS2 functions, too (i.e., \( DPQ \Leftrightarrow DP\cap QQ \)). As discussed in Keenan (2006), such a claim would be counter-exemplified by universal (e.g., *every*) and proportional (e.g., *most, half, one-third*) determiners. In this paper, we claim that these determiners fail to denote CONS2 functions in a specific way. CONS2 imposes a biconditional constraint on determiners and we can distinguish between two types of determiner denotations by separating each conditional statement that enters into its definition: (1) left-CONS2 functions defined by \( DPQ \Rightarrow DP\cap QQ \) and (2) right-CONS2 functions defined by \( DPQ \Leftarrow DP\cap QQ \). Universal and proportional determiners fail to denote CONS2 functions because they fail to denote right-CONS2 functions (*every* \( PQ \Leftrightarrow P\cap QQ \)) given that \( P \subseteq Q \Leftrightarrow P\cap Q \subseteq Q \). Such determiners do, however, denote left-CONS2 functions (*every* \( PQ \Rightarrow P\cap QQ \)) given that \( P \subseteq Q \Rightarrow P\cap Q \subseteq Q \).

We claim that NL-determiners denote left-CONS2 functions (the left-CONS2 Constraint).

In what follows, we take a closer look at determiners that seem to falsify the left-CONS2 Constraint. We show that, in each case, the offending inferences come not from the denotation of determiners but either from degree operators or from sentential operators such as \( \Box \).

1. Proportional determiners with an Upper Bound: \( \text{few}^{\text{PROP}} \) at most \( n \) and fewer than \( n \)

The context-dependent determiner \( \text{few} \) (as well as \( \text{many} \)) is ambiguous between cardinal and proportional readings (Partee, 1989, [\( ||\text{few}^{\text{CARD}}|| (P)/(Q) \Leftrightarrow |P\cap Q| < n \), a small number; \( ||\text{few}^{\text{PROP}}|| (P)/(Q) \Leftrightarrow |P\cap Q| / |P| < p \), a small proportion].) The proportional interpretation of \( \text{few} \) is problematic for the left-CONS2 Constraint (\( |P\cap Q| / |P| < p \Leftrightarrow |P\cap Q| \cap Q| / |P\cap Q| < p \)).

There is, however, reason to believe that the determiner \( \text{few} \) has a negative component that should be severed from the denotation of the determiner itself. In the presence of a modal operator, sentences with \( \text{few} \) has a (preferred) split-scope reading in which negation out-scopes the modal operator and the quantifier is interpreted in the scope of the modal operator (e.g., “They need few reasons to fire you.” SCOPE: \( \neg \Box \) many, see de Swart, 2000 and Solt, 2006). We assume that the proportional determiner \( \text{few} \) is a parametrized determiner with an additional degree argument (Hackl, 2000, Romero, 2015) and that it has the same denotation as the proportional determiner \( \text{many} \). [\( ||\text{many}^{\text{PROP}}|| = ||\text{few}^{\text{PROP}}|| = \lambda d. \lambda P. \lambda Q. |P\cap Q| / |P| \geq d \); McNally 1998]. Crucially, \( \text{many}^{\text{PROP}} \) denotes a left-CONS2 function for any \( d \). In analyzing split-scope readings, we assume that \( \text{few} \) must be licensed in the scope of Degree Negation (\( \lambda D.D' \)). Similar to adjectives, the determiner \( \text{few} \) is associated with a POS operator (\( \text{POS}_c = \lambda D_{\text{ext}}. \forall d \in \text{N}_c: D(d) \), Heim, 2006), which introduces a contextually determined Neutral Segment (an interval of degrees to be called \( \text{N}_c \)). Under these assumptions, the sentence in (1a) has the representation in (1b) and the denotation in (1c) (\( M_w = \lambda x. x \text{ is a mammal in } w', P_w = \lambda x. x \text{ survives in polar climate in } w' \))

(1) a. Few mammals can survive in the polar climate.
   b. [\( \text{POS}_c [\lambda \text{NEG} [\lambda d_1 [\text{can} (\lambda d_2 \text{few}^{\text{PROP}}_{\text{[D.NEG]} \text{mammals}} [\text{survive in polar climate}])]])]]
   c. \( \lambda w'. \forall d \in \text{N}_c, \forall w' \in \text{ACC}_{\text{w}}, [M_w\cap P_{w'}] / [M_w] < d \)

That is, by nomological necessity, the survival rate of mammals in the polar climate is less than the average of the survival rates of mammals in different climates (the negated de-dicto reading.)
The standard treatment of modified numerals within Generalized Quantifier Theory (GQT) is to analyze them as units (||at most one third|| = λP λQ. |P ∩ Q| / |P| ≤ 1/3, ||fewer than half|| = λP λQ. |P ∩ Q| / |P| < 1/2; Keenan & Stavi, 1986). Such entries are problematic for the left-CONS2 Constraint (|P ∩ Q| / |P| ≤ 1/3 ⇒ |P ∩ Q ∩ Q| / |P| ≤ 1/3). Recent work suggests that these determiners have internal parts that play a crucial role in their semantic composition. Building on Beck (2012), we take AT_MOST to denote the converse of degree subsethood: \( AT\_MOST(D)(D') \Leftrightarrow D' \subseteq D \). We take proportional numerals to denote degree segments (||70%|| = λd. 70/100 ≥ d, Takahashi, 2006, Solt, 2011). Following Nouwen (2010), we analyze proportional determiners as degree quantifiers. Under these assumptions, the sentence in (2a) has the representation in (2b) and the denotation in (2c).

(2) a. At most seventy percent of the students came to the party.
   b. \([\text{TP}_2 \ [AT\_MOST \ 70\%] \ λd_1 \ [\text{TP}_1 \ [\text{DP} \ d_1-\text{many}^{\text{PROP}} \ \text{students}]]) \ [\text{VP \ came \ to \ the \ party}]]\]
   c. \( \text{AT\_MOST}(λd.70/100 \geq d)(λd.\{|S\cap C| / |S| \geq d\} = 1 \iff λd.\{|S\cap C| / |S| \geq d \ \subseteq λd.\ 70/100 \geq d \)}

Our analysis of negative comparatives such as \text{fewer than half}, which relies on the comparative operator and the decomposition of \text{few}^{\text{PROP}}, is similar, as we discuss in the talk. Every determiner involved in the interpretation of expressions with modified numerals is a left-CONS2 function.

2. Restricted Universals (Exceptives and Approximatives)

In one of the earlier treatments of exceptives within GQT, Keenan and Stavi (1986) analyze the string \text{every...but John} as a determiner (\text{every...but John}(P)(Q) \Rightarrow P - Q = \{\text{John}'\}). This denotation is problematic for the left-CONS2 Constraint (\text{P - Q = \{\text{John}'\}} \not\Rightarrow (P ∩ Q - Q = \{\text{John}'\}). Building on Keenan & Stavi (1986), von Fintel (1993) argues that there are two components to the denotation of an exceptive determiner such as \text{every...but John}. The subtraction component is responsible for restricting the domain of quantification of the determiner. The exhaustivity component requires that the set that consists of the excepted entities be the smallest set whose exclusion renders the sentence true.

(3) \| \text{Det P but X Q} \| \Leftrightarrow \text{Det}(P - X)(Q) \quad (\text{Subtraction})
   \& \forall Y: Y \notin X \Rightarrow \neg\text{Det}(P - Y)(Q) \quad (\text{Exhaustivity})

Crnič (2018) claims that this subtraction, but not exhaustivity, is encoded in the meaning of exceptives (see also Gajewski, 2013 and Hirsh, 2016). Crnič suggests that VP-ellipsis constructions like (4b), where \text{strike-through} represents the elided material, pose a challenge for approaches that take the exhaustive inferences associated with exceptives to be internal and integral to the denotation of such determiners.

(4) a. In the exam, John solved every exercise but the last one.
   b. (To get an A), he really had to solve every exercise but the last one.

(5) (To get an A), John had to not solve the last exercise.

Due to \text{Condition on VP-Ellipsis}, which requires semantic parallelism between an ellipsis site and its antecedent, an integral approach to exceptives predicts (4b) to entail (5). (The details of this claim will be discussed in the talk). However, this prediction is not borne out. On the basis of this observation, Crnič (2018) claims that exceptives, as well as approximatives, which are identical to exceptives in their behavior in the context of \text{VP-ellipsis}, have subtractive analyses in (6):

(6) a. \| \text{every… but John} \|(P)(Q) \Leftrightarrow P - \{j\} \subseteq Q
   b. \| \text{almost every} \|(P)(Q) \Leftrightarrow \exists X: P - X \subseteq Q \quad (\text{Presupposition}: X \text{ is a relatively small set.})

Under these analyses, both \text{every… but John} and \text{almost every} denote left-CONS2 functions. The exhaustive inferences associated with exceptives and approximatives come from the obligatory presence of the Exh operator at the sentence level as in (7). We discuss this analysis in our talk.

(7) \[ \text{Exh} \ [\text{John had to solve every exercise but [the last one]}] \]
Conclusion: Once we do justice to the contribution of sentential and degree operators, we find that NL-determiners denote left-CONS2 functions after all.