Limited variable-force modals and the interaction of scalar,less implicatures

I show that exhaustification-based accounts of scalar implicatures, when combined with domain-restriction accounts of weak necessity modals, to directly and freely account for an otherwise puzzling and subtle pattern of variable-force modals in Kinande (Bantu J, DRC) without the need for additional stipulations within the semantic system.

**Limited variable-force** Depending on its context, the Kinande modal prefix *anga* can be interpreted as a possibility modal or as a weak necessity one, but never as a strong necessity modal (translations below from epistemic contexts):

(1) Kabunga a-**anga**-na-sya oko kalhasi ko munabwire
    Kabunga 3SG-MOD-T-come PREP class PREP today
    “Kabunga might come to class today”
    “Kambale should be coming to class today”
    #“Kabunga must be coming to class today”

Variable-force modals are previously attested in the Salish (Rullmann et al., 2008), Sahaptian (Deal, 2011), and Tukanoan (Jeretič, 2020) language families, among others; but this is the first attestation of a language with **limited** variable-force: Kinande *anga* does not span the entire scale of possibility to necessity, like the modals in prior accounts do, but instead it only spans a part of that scale, from possibility to weak necessity. Strong necessity is typically expressed with the modal verb *paswa* (‘must’). The pattern of ambiguity persists under downward entailing operators, except for clausemate negation.

This pattern is puzzling on prior analyses for a variety of reasons: If the modal variation is from domain restriction (e.g., Rullmann et al.), we would expect *anga* to remain ambiguous under clausemate negation, and perhaps also be compatible with strong necessity interpretations. If we analyze *anga* as an underlyingly weak modal and interpret the stronger readings as a lack of scalar implicature due to a missing strong modal (e.g., Deal), then the ambiguity under most downward entailing operators is unexpected. If we take a grammaticalized approach to scalar implicatures (Chierchia et al., 2010), as in Jeretič (2020) and Bowler (2014), exhaustifying over the subdomain alternatives of the modal base, a puzzle arises with regard to *anga*’s interaction with *paswa*: the presence of a strong necessity modal should create a scalar implicature and prevent any strengthening of *anga* via exhaustification; alternatively, if we exclude *paswa* from the scalar alternatives of *anga*, exhaustification would then strengthen *anga* all the way to a strong necessity interpretation, which is unacceptable. Instead we only see strengthening to weak necessity.

**Exhaustification gets us partway** A (potentially recursive) exhaustification operator over *anga*’s subdomain alternatives (the possible subsets of the accessible worlds), if *paswa* (‘must’) weren’t a scalar alternative, would account for most of *anga*’s distribution. Briefly, recursively exhausting over the subdomain alternatives of a weak expression that has no scalar alternatives derives a kind of strengthening: if we apply *exh* twice to (*p ∨ q*), if there is no scalar alternatives of (*p ∧ q*), we derive scalar implicatures (or rather, scaleless implicatures) of “not just *p*, and not just *q*”, and concatenated with the original proposition, this evaluates to the logical equivalent (*p ∧ q*) (Bowler, 2014). Similarly, such exhaustification over the subdomain alternatives of a modal that lacks a strong alternative will result in the logical equivalent of a universal quantifier over the same domain of modals (Jeretič, 2020). If the *exh* operator is taken to optionally applied at clause boundaries, then we derive the ambiguity of these expressions even in downward-entailing environments.
But this kind of exhaustification analysis fails to account for Kinande’s pattern: Kinande has a strong necessity modal paswa that appears to limit how far strengthening nga can go. Since paswa provides a true scalar alternative, we shouldn’t see any strengthening of nga from “can” to “should”. The only scalar implicature we should draw is one of “not have to”, but we see both such a scalar implicature and a scaleless implicature strengthening nga partway up the scale. Exhaustifying nga’s subdomain alternatives should strengthen nga-expressions directly to strong necessity (universal quantification over the domain of accessible worlds), or nga should remain a weak existential quantifying modal; we do not expect this partial strengthening.

**Domain restriction** Looking to the literature on weak necessity modals presents a solution: von Fintel & Iatridou (2008) and others have analyzed weak necessity modals like ought and should as universal quantifiers over an extra-restricted set of accessible worlds as compared to their strong necessity counterparts. While Kinande has neither a dedicated weak necessity modal nor one derived from strong necessity, nga does acquire a meaning much like this when it is strengthened. Note also that domain restriction of an existential quantifier/possibility modal does not result in a weakened reading, and in general is largely indiscernible in context; if the set of permissible worlds is very small and some $p$ is still true in at least one of those worlds, we are not much closer to saying that $p$ is obligatory; rather is it simply a very innocuous proposition. As a result, if we restrict nga’s domain we will only see any effect when nga is behaving as a universal quantifier, but not when it acts as an existential one.

Taking all this together, I define paswa as a strong necessity modal, and nga as a domain-restricted possibility modal, here following von Fintel & Iatridou in using multiple ordering source variables:

(2) $[\text{anga}(p)/w] = \exists w' [\text{Best}(\text{O}_2(\text{Best}(\text{O}_1(\text{MB}(w)))))(w') \& p(w')]$

(3) $[\text{paswa}(p)/w] = \forall w' [\text{Best}(\text{O}_1(\text{MB}(w)))(w') \rightarrow p(w')]$

**Putting it together** Given the definitions above, exhaustifying (either recursively or exhaustifying once with Innocent Inclusion as in Bar-Lev & Fox, 2020) over an nga-expression behaves exactly in line with what we see empirically: We draw a scalar implicature negating the paswa version of the expression via Innocent Exclusion, and then Innocent Inclusion on the subdomain alternatives of nga strengthens nga from an existential quantifier to a universal with a restricted domain, keeping it weaker than paswa.

(4) a. $\text{Best}(\text{O}_1(\text{MB}(w))) = \{w_1, w_2, w_3\}$  $\text{Best}(\text{O}_2(\text{Best}(\text{O}_1(\text{MB}(w)))))) = \{w_1, w_2\}$  
   b. $\text{exh}(\text{anga}(p)) = \exists w' [w' \in \{w_1, w_2\} \& p(w')] \& \neg \forall w' [w' \in \{w_1, w_2, w_3\} \rightarrow p(w')]$
   c. $\text{exh}(\text{exh}(\text{anga}(p))) = \exists w' [w' \in \{w_1\} \& p(w')] \& \exists w' [w' \in \{w_2\} \& p(w')]$
   $\Rightarrow \forall w' [w' \in \{w_1, w_2\} \rightarrow p(w')] \& \neg \forall w' [w' \in \{w_1, w_2, w_3\} \rightarrow p(w')]$

This result falls out naturally from our existing mechanisms for scalar implicature and weak necessity modals, coupled with the fact that Kinande lacks a dedicated weak necessity modal. The empirical and typological facts are accounted for, and a gap in the typology of variable-force modals is filled without overgenerating.